

Meta-Stable Brane Configurations of Triple Product Gauge Groups

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Abstract

From an $\mathcal{N} = 1$ supersymmetric electric gauge theory with the gauge group $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ with fundamentals for each gauge group and the bifundamentals, we apply Seiberg dual to each gauge group and obtain the $\mathcal{N} = 1$ supersymmetric dual magnetic gauge theories with dual matters including the additional gauge singlets. By analyzing the F-term equations of the dual magnetic superpotentials, we describe the intersecting brane configurations of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of this gauge theory. We apply also to the case for $\mathcal{N} = 1$ supersymmetric electric gauge theory with the gauge group $Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$ with flavors for each gauge group and the bifundamentals. Finally, we describe the meta-stable brane configurations of multiple product gauge groups.

1 Introduction

The nonsupersymmetric meta-stable vacua exist in $\mathcal{N} = 1$ SQCD with massive fundamental quarks where the masses are much smaller than the dynamical scale of the gauge sector [1]. The supersymmetry is broken by the rank condition and the classical flat directions can be lifted by quantum corrections which generate positive mass terms for the pseudomoduli leading to long-lived meta-stable vacua. When the direction which was not properly lifted by the one-loop potential occurs, by adding a set of singlets, one gets meta-stable vacua in the quiver gauge theory on fractional branes [2]. Other possible embedding by brane probes wrapping cycles of local Calabi-Yau and corresponding quiver gauge theories have inequivalent meta-stable vacua [3]. See the review paper [4] for the recent developments of dynamical supersymmetry breaking.

The geometrical approach for these meta-stable vacua, using the brane configuration in type IIA string theory, has many interesting features [5, 6, 7]. Turning on the quark masses in the electric theory corresponds to deform the superpotential by adding a term in linear in a singlet field in the dual magnetic theory. This is equivalent to move D6-branes in particular directions in type IIA brane configuration. The misalignment of D4-branes connecting NS5'-brane can be analyzed as a nontrivial F-term conditions providing nonzero vacuum expectation values of dual quarks. The fact that some of D4-branes connecting NS5'-brane can move in other two directions freely is exactly the classical moduli space of nonsupersymmetric vacua. See the review paper [8] for the gauge theory and the brane dynamics.

When we add an adjoint matter field into above $\mathcal{N} = 1$ SQCD with massive fundamental quarks, a meta-stable nonsupersymmetric long-lived vacuum was found by considering an addition of gauge singlet terms [9]. The corresponding brane configuration for these nonsupersymmetric meta-stable vacua was studied by realizing these deformation terms in the magnetic dual theory geometrically [10]. The lifting of the tree-level supersymmetry breaking brane configuration to M-theory was described in [7] by computing the equations of motion for the minimal area nonholomorphic curves. The behavior at infinity of this nonsupersymmetric brane configuration was different from that of the standard supersymmetric ground state of MQCD. The M-theory lift for symplectic and orthogonal gauge groups by introducing an orientifold 4-plane to the brane configuration for unitary gauge group was presented in [11].

When we add a symmetric flavor and a conjugate symmetric flavor to an $\mathcal{N} = 1$ SQCD with massive fundamental quarks, a meta-stable nonsupersymmetric long-lived vacuum was found by adding an orientifold 6-plane to the above brane configuration [12]. Moreover,

when an antisymmetric flavor, a conjugate symmetric flavor, and eight fundamental flavors are added to the $\mathcal{N} = 1$ SQCD with massive fundamental quarks, the nonsupersymmetric minimal energy brane configuration was obtained also in [13]. In these constructions, the precise brane motion was crucial to arrive at the meta-stable brane configurations in order to truncate the unwanted meson fields in the magnetic theory.

Then it is natural to ask what happens when we increase the number of gauge group. When a bifundamental is added to the $\mathcal{N} = 1$ product gauge theory with fundamentals for each gauge group, the nonsupersymmetric meta-stable brane configuration [14] can be constructed. Either the Seiberg dual for the first gauge group or for the second gauge group gave us the same meta-stable brane configuration. Although the magnetic superpotential has many more terms, compared with the ISS model [1], the F-term analysis enabled us to obtain the meta-stable vacua. For the product of symplectic and orthogonal gauge groups, one needs to add an orientifold 4-plane as well as three NS-branes, two kinds of D4-branes and two kinds of D6-branes [15]. On the other hand, when we add an orientifold 6-plane to the brane configuration for product gauge theory with unitary groups, the matter contents will be different in general and the extra NS-branes should be included. In this case, the meta-stable brane configuration was described in [16].

When the D6-branes in above brane configurations are replaced by other NS5'-brane, the meta-stable vacua of [1] arises in some region of parameter space when the D4-branes and anti D4-branes can decay and the geometric misalignment of flavor D4-branes occurs [17]. Adding an orientifold 4-plane or orientifold 6-plane to the brane configuration of [17] implies that the gauge group is a product of a symplectic group and an orthogonal group or a product of unitary groups. Then the geometric misalignment of flavor D4-branes also occurs [18]. Further generalization by using more NS-branes, orientifold 4-plane, or orientifold 6-plane was obtained from the recent works [19] where there exists a triple product of gauge groups with bifundamentals and [20] where other extra matter contents are present.

In this paper, we continue to study for the meta-stable brane configuration in the context of triple product gauge groups. Compared with two product gauge groups studied in [14, 15], the Seiberg dual for the middle gauge group has a new feature in the sense that the ranks of the first gauge group and the third gauge group appear in the number of dual colors for the middle gauge group and there exist two possible magnetic brane configurations where the magnetic superpotentials have different form and also the different matter contents are present. On the other hand, the Seiberg duals for the first gauge group and the third gauge group look similar to the ones in two product gauge groups because when we take the Seiberg dual for the first(third) gauge group, the fields for the third(first) gauge group in the magnetic

theory are the same as those for the third(first) gauge group in the electric theory.

One can easily generalize the meta-stable brane configuration to the ones corresponding to a multiple product of gauge groups. Then one takes the Seiberg dual for the first gauge group factor, the last gauge group factor and for any gauge group factor except the first and last gauge group factors. One can write down the magnetic superpotentials in terms of the cubic interactions between the gauge singlets and dual matters.

We also add an orientifold 4-plane to the brane configurations consisting of four NS-branes, three kinds of D4-branes and three kinds of D6-branes in type IIA string theory. Then the gauge group will be a product of symplectic and orthogonal gauge groups alternatively. The meta-stable brane configuration for the multiple product gauge groups is also discussed.

In section 2, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with fundamentals and bifundamentals, and deform this theory by adding the mass term for the quarks for each gauge group. Then we construct the Seiberg dual magnetic theories for each gauge group factor with corresponding dual matters as well as additional gauge singlets, by brane motion and linking number counting. Finally, the nonsupersymmetric brane configurations are found by recombination and splitting for the flavor D4-branes. One generalizes to the meta-stable brane configurations corresponding to a multiple product of gauge groups and describe them very briefly.

In section 3, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$ gauge theory with fundamentals, vectors, and bifundamentals, and deform this theory by adding the mass term for the quarks for each gauge group. In the brane configuration, this can be obtained from the brane configuration given in section 2 by inserting the appropriate orientifold 4-planes. Then we construct the Seiberg dual magnetic theories for each gauge group factor with corresponding dual matters as well as extra gauge singlets, by brane motion and linking number counting. Finally, the nonsupersymmetric brane configurations are found. These can be seen from those brane configurations in section 2 by the action of orientifold 4-plane. The generalization to a multiple product of gauge groups is described. Compared with the ones in section 2, in this case, there exist more magnetic dual theories because the gauge group factors have both symplectic and orthogonal gauge groups.

Finally, in section 4, we summarize what we have found in this paper and make some comments for the future directions ¹.

¹Some different directions on the meta-stable vacua are present in recent relevant works [21]-[45] where some of them are described in the type IIB string theory. These are not complete list. It would be very interesting to find out how the meta-stable brane configurations from type IIA string theory including the

2 Nonsupersymmetric meta-stable brane configurations of $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ and its multiple product gauge theories

For the meta-stable brane configurations, it is necessary to have nonzero masses for the quarks (corresponding to relative displacement between D6-branes and D4-branes in type IIA string theory) and to take the Seiberg dual theory (corresponding to the magnetic theory obtained from the electric theory via brane motion or field theory analysis) [1]. We need to understand the brane configurations both from the electric theory (where one has mass-deformed superpotential) and from the magnetic theory (where the meta-stable states are long-lived parametrically). There exist four possible magnetic brane configurations for the triple product gauge group $SU(N_c) \times SU(N'_c) \times SU(N''_c)$, depending on whether the dual gauge group we take is the first gauge group, the second gauge group (in which there exist two magnetic brane configurations), or the third gauge group.

After reviewing the electric brane configuration, we describe the four magnetic brane configurations, and then the nonsupersymmetric brane configurations are found by recombination of some flavor D4-branes and color D4-branes and splitting procedure between those flavor D4-branes and the remnant of flavor D4-branes.

2.1 Electric theory

The gauge group is given by $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ and the matter contents [46, 47] are given by

- N_f chiral multiplets Q are in the fundamental representation under the $SU(N_c)$, N_f chiral multiplets \tilde{Q} are in the antifundamental representation under the $SU(N_c)$ and then Q are in the representation $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$ while \tilde{Q} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1}, \mathbf{1})$ under the whole gauge group
- N'_f chiral multiplets Q' are in the fundamental representation under the $SU(N'_c)$, N'_f chiral multiplets \tilde{Q}' are in the antifundamental representation under the $SU(N'_c)$ and then Q' are in the representation $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ while \tilde{Q}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{1})$ under the whole gauge group
- N''_f chiral multiplets Q'' are in the fundamental representation under the $SU(N''_c)$, N''_f chiral multiplets \tilde{Q}'' are in the antifundamental representation under the $SU(N''_c)$ and then Q'' are in the representation $(\mathbf{1}, \mathbf{1}, \mathbf{N}''_c)$ while \tilde{Q}'' are in the representation $(\mathbf{1}, \mathbf{1}, \overline{\mathbf{N}}''_c)$ under the

present work are related to those brane configurations from type IIB string theory.

whole gauge group

- The flavor-singlet field F is in the bifundamental representation $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$ under the whole gauge group and its complex conjugate field \tilde{F} is in the bifundamental representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c, \mathbf{1})$ under the whole gauge group

- The flavor-singlet field G is in the bifundamental representation $(\mathbf{1}, \mathbf{N}'_c, \overline{\mathbf{N}}''_c)$ under the whole gauge group and its complex conjugate field \tilde{G} is in the bifundamental representation $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{N}''_c)$ under the whole gauge group

This is a simple generalization to triple product gauge groups from the two product gauge groups [48, 49, 46, 47]. Then the gauge group and matter contents we consider are summarized as follows:

gauge group :	$SU(N_c) \times SU(N'_c) \times SU(N''_c)$	
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$

In the electric theory, since there exist N_f quarks Q , N_f quarks \tilde{Q} , one bifundamental field F (which will give rise to the contribution of N'_c), and its complex conjugate field \tilde{F} (which will give rise to the contribution of N'_c), the coefficient of the beta function of the first gauge group factor is $b_{SU(N_c)} = 3N_c - N_f - N'_c$. Similarly, since there exist N'_f quarks Q' , N'_f quarks \tilde{Q}' , one bifundamental field F (which gives the contribution of N_c), its complex conjugate field \tilde{F} (which will give the contribution of N_c), one bifundamental field G (which will give rise to the contribution of N''_c), and its complex conjugate field \tilde{G} (which will give the contribution of N''_c), the coefficient of the beta function for the second gauge group factor is given by $b_{SU(N'_c)} = 3N'_c - N'_f - N_c - N''_c$. Finally, since there exist N''_f quarks Q'' , N''_f quarks \tilde{Q}'' , one bifundamental field G (which will give rise to the contribution of N'_c), and its complex conjugate field \tilde{G} (which will give rise to the contribution of N'_c), the coefficient of the beta function of the third gauge group factor can be read off $b_{SU(N''_c)} = 3N''_c - N''_f - N'_c$. There is a symmetry between $b_{SU(N_c)}$ and $b_{SU(N''_c)}$ such that the former becomes the latter by replacing N_c and N_f with N''_c and N''_f respectively.

The anomaly free global symmetry contains $[SU(N_f) \times SU(N'_f) \times SU(N''_f)]^2 \times U(1)_R$ [46] and let us denote the strong coupling scales for $SU(N_c)$ as Λ_1 , for $SU(N'_c)$ as Λ_2 and for $SU(N''_c)$ as Λ_3 respectively. The electric theory is asymptotically free when $b_{SU(N_c)} > 0$ for the

$SU(N_c)$ gauge theory, when $b_{SU(N'_c)} > 0$ for the $SU(N'_c)$ gauge theory, and when $b_{SU(N''_c)} > 0$ for the $SU(N''_c)$ gauge theory.

The classical electric superpotential is

$$W_{elec} = \left(\mu A^2 + \lambda Q A \tilde{Q} + \tilde{F} A F + \mu' A'^2 + \lambda' Q' A' \tilde{Q}' + \tilde{F} A' F + \tilde{G} A' G \right. \\ \left. + \mu'' A''^2 + \lambda'' Q'' A'' \tilde{Q}'' + \tilde{G} A'' G \right) + m Q \tilde{Q} + m' Q' \tilde{Q}' + m'' Q'' \tilde{Q}'' \quad (2.1)$$

where the coefficient functions $\mu, \mu', \mu'', \lambda, \lambda'$ and λ'' are given by six rotation angles for the branes in type IIA string theory [46, 50]. We do not write down the dependences on these angles explicitly. Each gauge group factor has two rotation angles on NS-brane and D6-branes. Here the adjoint field for $SU(N_c)$ gauge group is denoted by A , the adjoint field for $SU(N'_c)$ gauge group is denoted by A' , and the adjoint field for $SU(N''_c)$ gauge group is denoted by A'' . The mass terms of these adjoint fields are related to the rotation angles of NS-branes. The couplings of fundamentals with these adjoint fields are related also to the rotation angles of NS-branes as well as the rotation angles of D6-branes. We add the mass terms for each fundamental flavor. Setting the fields $Q'', \tilde{Q}'', G, \tilde{G}$ and A'' to zero, the superpotential becomes the one described in [46, 50, 14].

After integrating out the adjoint fields A, A' and A'' , this superpotential (2.1) at the particular orientations for branes, i.e., the case where any two neighboring NS-branes are perpendicular to each other, will reduce to the last three mass-deformed terms since the coefficient functions $\frac{1}{\mu}, \frac{1}{\mu'}$ and $\frac{1}{\mu''}$ vanish at these particular rotation angles for branes. It does not matter whether λ, λ' or λ'' vanishes because even if these coefficient functions are not zero, λ, λ' - or λ'' -dependent terms all vanish because they contain $\frac{1}{\mu}, \frac{1}{\mu'}$ - or $\frac{1}{\mu''}$ -prefactors. Then the classical superpotential by deforming the massless case by adding the mass terms for the quarks $Q(Q')[Q'']$ and $\tilde{Q}(\tilde{Q}')[\tilde{Q}'']$ is given by

$$W_{elec} = m Q \tilde{Q} + m' Q' \tilde{Q}' + m'' Q'' \tilde{Q}'' \quad (2.2)$$

When we discuss the meta-stable brane configurations for the particular dual gauge group with corresponding massive flavors, the other flavors corresponding to other two gauge groups will be massless.

The type IIA brane configuration for this mass-deformed theory can be described by as follows ². The N_c -color D4-branes (01236) are suspended between the $NS5_L$ -brane (012345) and the $NS5'_L$ -brane (012389) along x^6 direction, together with N_f D6-branes (0123789) which are parallel to $NS5'_L$ -brane and have nonzero v direction. The $NS5_R$ -brane is located at the

²Let us introduce two complex coordinates $v \equiv x^4 + ix^5$ and $w \equiv x^8 + ix^9$ for convenience.

right hand side of the $NS5'_L$ -brane along the x^6 direction and there exist N'_c -color D4-branes suspended between them, with N'_f D6-branes which have nonzero v direction. Moreover, the $NS5'_R$ -brane is located at the right hand side of the $NS5_R$ -brane along the x^6 direction and there exist N''_c -color D4-branes suspended between them, with N''_f D6-branes which have nonzero v direction ³.

We summarize the $\mathcal{N} = 1$ supersymmetric electric brane configuration in type IIA string theory as follows:

- $NS5_L(NS5_R)$ -brane in (012345) directions.
- $NS5'_L(NS5'_R)$ -brane in (012389) directions.
- $N_c(N'_c)[N''_c]$ -color D4-branes in (01236) directions.
- $N_f(N'_f)[N''_f]$ D6-branes in (0123789) directions.

Now we draw this electric brane configuration in Figure 1 and we put the coincident $N_f(N'_f)[N''_f]$ D6-branes in the nonzero v direction in general. The quarks $Q(Q')[Q'']$ and $\tilde{Q}(\tilde{Q}')[\tilde{Q}'']$ correspond to strings stretching between the $N_c(N'_c)[N''_c]$ -color D4-branes with $N_f(N'_f)[N''_f]$ D6-branes. The bifundamentals $F(G)$ and $\tilde{F}(\tilde{G})$ correspond to strings stretching between the $N_c(N'_c)$ -color D4-branes with $N'_c(N''_c)$ -color D4-branes.

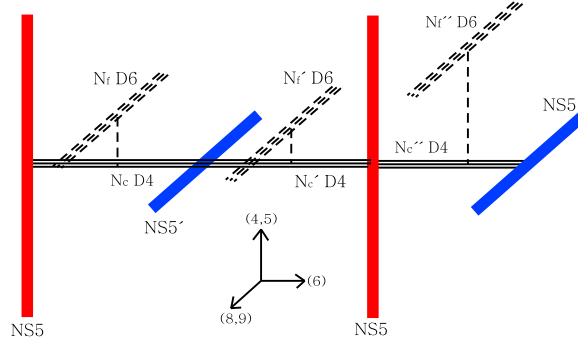


Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration with $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge group with fundamentals $Q(Q')[Q'']$ and $\tilde{Q}(\tilde{Q}')[\tilde{Q}'']$ for each gauge group and bifundamentals $F(G)$ and $\tilde{F}(\tilde{G})$. The two NS5-branes can be written as $NS5_{L,R}$ -branes while the two NS5'-branes can be denoted by $NS5'_{L,R}$ -branes.

³Basically, if there are no $NS5_L$ -brane, N_c D4-branes and N_f D6-branes, this brane configuration is exactly the same as the one in [14]. One can consider the other brane configuration by rotations of NS-branes in above brane configuration by $\frac{\pi}{2}$ angles. That is, $NS5_{L,R}$ -branes go to $NS5'_{L,R}$ -branes and vice versa. In other words, this other brane configuration can be obtained from the one in [14], by adding $NS5_R$ -brane, N''_c D4-branes and N''_f D6-branes to the right. Then we will see that all the meta-stable brane configurations from this other brane configuration can be obtained by viewing the Figures 2B, 3B, 4B and 5B from the negative w direction.

2.2 Magnetic theory with dual for third gauge group

One considers dualizing one of the gauge groups regarding as the other gauge groups as a spectator. In this subsection, one takes the Seiberg dual for the third gauge group factor $SU(N_c'')$ while remaining the first and second gauge group factors $SU(N_c)$ and $SU(N_c')$ unchanged. One ignores the dynamics of the other gauge groups. We consider the case where $\Lambda_3 \gg \Lambda_1, \Lambda_2$, in other words, the dualized group's dynamical scale is far above that of the other spectator groups.

Let us move the $NS5_R$ -brane to the right all the way past the $NS5'_R$ -brane. After this brane motion, one arrives at the Figure 2A. Note that there exists a creation of N_f'' D4-branes connecting N_f'' D6-branes and $NS5'_R$ -brane because the N_f'' D6-branes are perpendicular(or are not parallel) to the $NS5_R$ -brane in Figure 1 and we consider massless quarks for $Q(Q')$ and $\tilde{Q}(\tilde{Q}')$. The linking number [51] of $NS5_R$ -brane from Figure 2A can be read off and is given by $L_5 = \frac{N_f''}{2} - \tilde{N}_c''$. On the other hand, the linking number of $NS5_R$ -brane from Figure 1 is $L_5 = -\frac{N_f''}{2} + N_c'' - N_c'$. Due to the connection of N_c' D4-branes with $NS5_R$ -brane in Figure 1, the presence of N_c' in the linking number arises. This N_c' dependence will appear in many places later and is common feature when we discuss about the product gauge group. From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c'' = N_f'' + N_c' - N_c''. \quad (2.3)$$

Let us draw this magnetic brane configuration in Figure 2A and recall that we put the coincident N_f'' D6-branes in the nonzero v -direction in the electric theory and consider massless flavors for Q and Q' by putting N_f and N_f' D6-branes at $v = 0$. The N_f'' created D4-branes connecting between D6-branes and $NS5'_R$ -brane can move freely in the w -direction. Moreover, since N_c' D4-branes are suspending between two equal $NS5'_{L,R}$ -branes located at different x^6 coordinate, these D4-branes can slide along the w -direction also. If we ignore the $NS5_L$ -brane, N_c D4-branes, N_f D6-branes, the $NS5'_L$ -brane, N_c' D4-branes and N_f' D6-branes(detaching these branes from Figure 2A), then this brane configuration leads to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}_c'' = N_f'' - N_c'')$ with N_f'' massive flavors [3, 6, 7].

The IR dynamics of the electric theory is described by Seiberg dual and one uses the variables of dual theory instead of the original variables for the electric theory. Then the dual magnetic gauge group is given by $SU(N_c) \times SU(N_c') \times SU(\tilde{N}_c'')$ and the matter contents are given by

- N_f chiral multiplets Q are in the fundamental representation under the $SU(N_c)$, N_f chiral multiplets \tilde{Q} are in the antifundamental representation under the $SU(N_c)$ and then Q are in the representation $(\mathbf{N}_c, \mathbf{1}, \mathbf{1})$ while \tilde{Q} are in the representation $(\overline{\mathbf{N}}_c, \mathbf{1}, \mathbf{1})$ under the

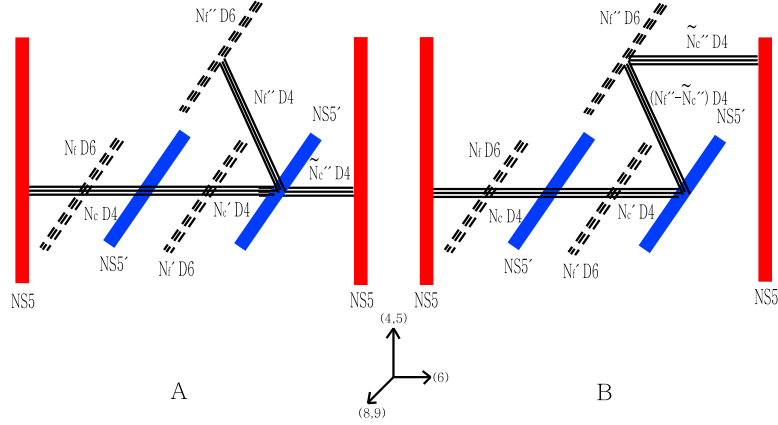


Figure 2: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'')$ gauge group with fundamentals $Q(Q')[q'']$ and $\tilde{Q}(\tilde{Q}')[\tilde{q}'']$ for each gauge group, bifundamentals $F(g)$ and $\tilde{F}(\tilde{g})$, and gauge singlets in Figure 2A. In Figure 2B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q(Q')$ and $\tilde{Q}(\tilde{Q}')$ is given.

whole gauge group

- N'_f chiral multiplets Q' are in the fundamental representation under the $SU(N'_c)$, N'_f chiral multiplets \tilde{Q}' are in the antifundamental representation under the $SU(N'_c)$ and then Q' are in the representation $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ while \tilde{Q} are in the representation $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{1})$ under the whole gauge group

- N''_f chiral multiplets q'' are in the fundamental representation under the $SU(\tilde{N}_c'')$, N''_f chiral multiplets \tilde{q}'' are in the antifundamental representation under the $SU(\tilde{N}_c'')$ and then q'' are in the representation $(\mathbf{1}, \mathbf{1}, \tilde{\mathbf{N}}_c'')$ while \tilde{q}'' are in the representation $(\mathbf{1}, \mathbf{1}, \overline{\tilde{\mathbf{N}}}_c'')$ under the whole gauge group

- The flavor-singlet field F is in the bifundamental representation $(\mathbf{N}_c, \overline{\mathbf{N}}'_c, \mathbf{1})$ under the whole gauge group and its complex conjugate field \tilde{F} is in the bifundamental representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c, \mathbf{1})$ under the whole gauge group

- The flavor-singlet field g is in the bifundamental representation $(\mathbf{1}, \mathbf{N}'_c, \overline{\tilde{\mathbf{N}}}_c'')$ under the whole gauge group and its complex conjugate field \tilde{g} is in the bifundamental representation $(\mathbf{1}, \overline{\mathbf{N}}'_c, \tilde{\mathbf{N}}_c'')$ under the whole gauge group

There are $(N''_f + N'_c)^2$ gauge singlets in the third gauge group factor as follows:

- N''_f -fields X' are in the fundamental representation under the $SU(N'_c)$, its complex conjugate N''_f -fields \tilde{X}' are in the antifundamental representation under the $SU(N'_c)$ and then X' are in the representation $(\mathbf{1}, \mathbf{N}'_c, \mathbf{1})$ under the whole gauge group while \tilde{X}' are in the representation $(\mathbf{1}, \overline{\mathbf{N}}'_c, \mathbf{1})$ under the whole gauge group

These additional N_f'' - $SU(N_c')$ fundamentals and N_f'' - $SU(N_c')$ antifundamentals are originating from the $SU(N_c'')$ chiral mesons GQ'' and $\tilde{G}\tilde{Q}''$ respectively where the color indices are contracted each other. Therefore, there are free indices for a single color and a single flavor. Then the strings stretching between the N_f'' D6-branes and N_c' D4-branes will give rise to these additional N_f'' - $SU(N_c')$ fundamentals and N_f'' - $SU(N_c')$ antifundamentals.

- $N_f''^2$ -fields M'' are in the representation $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the whole gauge group

This corresponds to the $SU(N_c'')$ chiral meson $Q''\tilde{Q}''$ where the color indices are contracted. It is clear to see that since the N_f'' D6-branes are parallel to the $NS5'_R$ -brane from Figure 2A, the newly created N_f'' -flavor D4-branes can slide along the plane consisting of these N_f'' D6-branes and $NS5'_R$ -brane freely. The fluctuations of the gauge-singlet M'' correspond to the motion of N_f'' flavor D4-branes along (789) directions in Figure 2A. As we will see later, for the nonsupersymmetric brane configuration, a misalignment for the N_f'' -flavor D4-branes arises and some of the vacuum expectation value of M'' is fixed and the remaining components are arbitrary.

- The $N_c'^2$ -fields Φ' is in the representation $(\mathbf{1}, \mathbf{N}_c'^2 - \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the whole gauge group

This corresponds to the $SU(N_c'')$ chiral meson $G\tilde{G}$ where the color indices for the third gauge group are contracted each other and note that G has a representation $(\mathbf{1}, \mathbf{N}_c', \overline{\mathbf{N}}_c'')$ of an electric theory while \tilde{G} has a representation $(\mathbf{1}, \overline{\mathbf{N}}_c', \mathbf{N}_c'')$ of an electric theory. The fluctuations of the singlet Φ' correspond to the motion of N_c' D4-branes suspended two $NS5'_{L,R}$ -branes along the (789) directions in Figure 2A. The fact that these D4-branes can slide along the w -direction implies that the vacuum expectation value of Φ is arbitrary.

In the dual theory, since there exist N_f'' quarks q'' , N_f'' quarks \tilde{q}'' , one bifundamental field g (which will give rise to the contribution of N_c'), and its complex conjugate field \tilde{g} (which will give rise to the contribution of N_c'), the coefficient of the beta function for the third gauge group factor is $b_{SU(\tilde{N}_c'')}^{mag} = 3\tilde{N}_c'' - N_f'' - N_c' = 2N_f'' + 2N_c' - 3N_c''$ where we inserted the number of colors given in (2.3) in the second equality. Since there exist N_f' quarks Q' , N_f' quarks \tilde{Q}' , one bifundamental field g (which will give rise to the contribution of \tilde{N}_c''), its complex conjugate field \tilde{g} (which will give rise to the contribution of \tilde{N}_c''), N_f'' -fields X' , its complex conjugate N_f'' -fields \tilde{X}' , the singlet Φ' (which will give rise to N_c'), one bifundamental field F (which will give rise to the contribution of N_c), and its complex conjugate field \tilde{F} (which will give rise to the contribution of N_c), the coefficient of the beta function of second gauge group factor is $b_{SU(N_c')}^{mag} = 3N_c' - N_f' - \tilde{N}_c'' - N_f'' - N_c' - N_c = N_c' + N_c'' - N_c - 2N_f'' - N_f'$. Similarly, since there exist N_f quarks Q , N_f quarks \tilde{Q} , one bifundamental field F (which will give rise to the contribution of N_c'), and its complex conjugate field \tilde{F} (which will give rise to

the contribution of N'_c , the coefficient of the beta function of the first gauge group factor is $b_{SU(N_c)}^{mag} = 3N_c - N_f - N'_c = b_{SU(N_c)}$. Note that the $SU(N_c)$ fields in the magnetic theory are the same as those of the electric theory.

Therefore, $SU(N_c)$, $SU(N'_c)$ and $SU(\tilde{N}_c'')$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One relies on the perturbative calculations at low energy for this magnetic IR free region $b_{SU(N_c)}^{mag} < 0$, $b_{SU(N'_c)}^{mag} < 0$ and $b_{SU(\tilde{N}_c'')}^{mag} < 0$. Note that the $SU(N'_c)$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SU(N'_c)} - b_{SU(N'_c)}^{mag} > 0$, $SU(N'_c)$ is more asymptotically free than $SU(N'_c)^{mag}$. Neglecting the $SU(N_c)$ and $SU(N'_c)$ dynamics, the magnetic $SU(\tilde{N}_c'')$ is IR free when

$$N_c'' - N'_c < N_f'' < \frac{3}{2}N_c'' - N'_c.$$

Instead of $SU(N_c) \times SU(N'_c) \times SU(N_c'')$ gauge theory, by performing the dualizing the third gauge group, we have an $SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'')$ gauge theory with additional fields X' , \tilde{X}' , M'' and Φ' and the gauge group and matter contents are summarized as follows:

	gauge group :	$SU(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'')$
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$q''_{f''} \oplus \tilde{q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$F \oplus \tilde{F}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$g \oplus \tilde{g}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$(X'_{n''} \equiv) GQ'' \oplus \tilde{G}\tilde{Q}'' (\equiv \tilde{X}'_{\tilde{n}''})$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (n'', \tilde{n}'' = 1, \dots, N''_f)$
	$(M''_{f'', \tilde{g}''} \equiv) Q''\tilde{Q}''$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f'', \tilde{g}'' = 1, \dots, N''_f)$
	$(\Phi' \equiv) G\tilde{G}$	$(\mathbf{1}, \mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The dual magnetic superpotential, by taking the mass term (2.2) for Q'' and \tilde{Q}'' and massless flavors for $m = m' = 0$ in the electric theory, which is equal to put a linear term in M'' in the dual magnetic theory, in addition to the cubic interaction terms, takes the form

$$W_{dual} = \left(M'' q'' \tilde{q}'' + g X' q'' + \tilde{g} \tilde{q}'' \tilde{X}' + \Phi' g \tilde{g} \right) + m'' M''.$$

Here q'' and \tilde{q}'' are fundamental and antifundamental for the third gauge group index respectively. Then, $q'' \tilde{q}''$ has rank \tilde{N}_c'' while m'' has a rank N_f'' . Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M'' , cannot be satisfied if the rank N_f'' exceeds \tilde{N}_c'' . This is so-called rank condition and the supersymmetry is broken [14]. Other

F-term equations are satisfied by taking the vacuum expectation values of g, \tilde{g}, X' and \tilde{X}' to vanish.

More explicitly, the classical moduli space of vacua can be obtained from the following F-term equations

$$\begin{aligned} q''\tilde{q}'' + m &= 0, & \tilde{q}''M'' + X'g &= 0, \\ M''q'' + \tilde{g}\tilde{X}' &= 0, & X'q'' + \tilde{g}\Phi' &= 0, \\ q''g &= 0, & \tilde{q}''\tilde{X}' + \Phi'g &= 0, \\ \tilde{g}\tilde{q}'' &= 0, & g\tilde{g} &= 0. \end{aligned}$$

Then, it is easy to see that there exist three reduced equations

$$\tilde{q}''M'' = 0 = M''q'', \quad q''\tilde{q}'' + m'' = 0$$

and other F-term equations are satisfied if one takes the zero vacuum expectation values for the fields g, \tilde{g}, X' and \tilde{X}' . Then the solutions for the equations can be written as follows:

$$\begin{aligned} \langle q'' \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}_c''} \\ 0 \end{pmatrix}, \quad \langle \tilde{q}'' \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}_c''} & 0 \end{pmatrix}, \quad \langle M'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_0'' \mathbf{1}_{N_f'' - \tilde{N}_c''} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0. \end{aligned}$$

Let us expand around a point on these vacua, as done in [1]. Then the remaining relevant terms of superpotential can be written as

$$W_{dual}^{rel} = M_0'' (\delta\varphi \delta\tilde{\varphi} + m'') + \delta Z \delta\varphi \tilde{q}_0'' + \delta\tilde{Z} q_0'' \delta\tilde{\varphi}$$

by following the same fluctuations for the various fields as in [12]:

$$\begin{aligned} q'' &= \begin{pmatrix} q_0'' \mathbf{1}_{\tilde{N}_c''} + \frac{1}{\sqrt{2}}(\delta\chi_+ + \delta\chi_-) \mathbf{1}_{\tilde{N}_c''} \\ \delta\varphi \end{pmatrix}, \quad \tilde{q}'' = \begin{pmatrix} \tilde{q}_0'' \mathbf{1}_{\tilde{N}_c''} + \frac{1}{\sqrt{2}}(\delta\chi_+ - \delta\chi_-) \mathbf{1}_{\tilde{N}_c''} & \delta\tilde{\varphi} \end{pmatrix}, \\ M'' &= \begin{pmatrix} \delta Y & \delta Z \\ \delta\tilde{Z} & M_0'' \mathbf{1}_{N_f'' - \tilde{N}_c''} \end{pmatrix} \end{aligned}$$

as well as the fluctuations of g, \tilde{g}, X' and \tilde{X}' . Note that there exist also three kinds of terms, the vacuum $\langle \tilde{q}'' \rangle$ multiplied by $\delta\tilde{g}\delta\tilde{X}'$, the vacuum $\langle q'' \rangle$ multiplied by $\delta X'\delta g$, and the vacuum $\langle \Phi' \rangle$ multiplied by $\delta g\delta\tilde{g}$. However, by redefining these, they do not enter the contributions for the one loop result, up to quadratic order. As done in [52], it turns out that $m_{M_0''}^2$ will contain $(\log 4 - 1) > 0$ which implies that these vacua are stable.

Now let us recombine \tilde{N}_c'' flavor D4-branes among N_f'' flavor D4-branes(connecting between D6-branes and $NS5'_R$ -brane) with the same number of color D4-branes(connecting between $NS5'_R$ -brane and $NS5_R$ -brane) and push them in $+v$ direction from Figure 2A. We assume that $N_c'' \geq N_c'$. After this procedure, there are no color D4-branes between $NS5'_R$ -brane and $NS5_R$ -brane. For the flavor D4-branes, we are left with only $(N_f'' - \tilde{N}_c'') = N_c'' - N_c'$ flavor D4-branes connecting between D6-branes and $NS5'_R$ -brane.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 2B. If we ignore the $NS5'_L$ -brane, N_c' D4-branes, N_f' D6-branes, $NS5_L$ -brane, N_c D4-branes and N_f D6-branes(detaching these from Figure 2B), as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}_c'' = N_f'' - N_c'')$ with N_f'' massive flavors [3, 6, 7].

The nonsupersymmetric minimal energy brane configuration in Figure 2B leads to the Figure 3 of [14] if we ignore the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes. Moreover, the Figure 2B with a replacement N_f'' D6-branes by the $NS5'$ -brane(neglecting the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes) will become the Figure 2B of [19] together with a reflection with respect to the $NS5_L$ -brane and a rotation of $NS5_R$ -brane by $\frac{\pi}{2}$ angle.

When one moves the $NS5'_R$ -brane to the left all the way past the $NS5_R$ -brane, as long as the N_f'' D6-branes are not parallel to $NS5'_R$ -brane in an electric theory, then one arrives at the other magnetic brane configuration similar to the Figure 2. The only difference is that the N_f'' D6-branes are located at the right hand side of the $NS5_R$ -brane.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, and moving the $NS5_R$ -brane to the left, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 5B(which will appear later) with a reflection with respect to the $NS5$ -brane. Furthermore, by moving the $NS5'_L$ -brane to the right (as long as the N_f D6-branes are not parallel to $NS5'_L$ -brane in an electric theory), one gets the nonsupersymmetric minimal energy brane configuration which is exactly the new figure in previous paragraph with a reflection with respect to the $NS5_L$ -brane.

After lifting the type IIA description to M-theory, the corresponding magnetic M5-brane configuration, in a background space of $xt = v^{N_f + N_f'} \prod_{k=1}^{N_f''} (v - e_k)$ where e_k is the position of the N_f'' D6-branes in the v direction and this four dimensional space replaces (45610) directions, is described by [53]

$$t^4 + (v^{N_c} + \dots)t^3 + (v^{N_c + N_f} + \dots)t^2 + (v^{\tilde{N}_c'' + 2N_f + N_f'} + \dots)(v - m)^{N_f''}t + v^{3N_f + 2N_f'}(v - m)^{2N_f''} = 0$$

where we have ignored the lower power terms in v in t^3, t^2 and t and the scales for the gauge groups in front of the first term and the last term, for simplicity. For fixed x , the coordinate t corresponds to y .

From this curve of quartic equation for t above, the asymptotic regions for four NS5-branes can be classified by looking at the first two terms providing $NS5_L$ -brane asymptotic region, next two terms providing $NS5'_L$ -brane asymptotic region, next two terms providing $NS5'_R$ -brane asymptotic region, and the last two terms giving $NS5_R$ -brane asymptotic region as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{N_c} + \dots & NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{N_f+N'_f+N''_f-\tilde{N}_c''} + \dots & NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{N'_c-N_c+N_f} + \dots & NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{\tilde{N}_c''-N'_c+N_f+N'_f+N''_f} + \dots & NS'_R \text{ asymptotic region.} \end{aligned}$$

Here the two $NS5'_{L,R}$ -branes are moving in the $+v$ direction holding everything else fixed instead of moving N''_f D6-branes in the $+v$ direction, as in [7]. The harmonic function sourced by the D6-branes can be written explicitly by summing over three contributions from the N_f , N'_f and N''_f D6-branes and similar analysis to both solve the differential equation and find out the nonholomorphic curve can be done. We expect that an instability from a new M5-brane mode arises.

2.3 Magnetic theory with dual for second gauge group

Based on the procedure in previous subsection, one continues to consider dualizing one of the gauge groups regarding as the other gauge groups as a spectator. One takes the Seiberg dual for the second gauge group factor $SU(N'_c)$ while remaining the first and third gauge group factor $SU(N_c)$ and $SU(N''_c)$ unchanged. Also, since the dualized group's dynamical scale is far above that of the other spectator groups, we consider the case where $\Lambda_2 \gg \Lambda_1, \Lambda_3$.

Let us move the $NS5_R$ -brane in Figure 1 to the left all the way past the $NS5'_L$ -brane. There is also a possibility to move $NS5'_L$ -brane to the right instead. We'll come to this next subsection. After this brane motion, one arrives at the Figure 3A. The linking number of $NS5_R$ -brane from Figure 3A is $L_5 = -\frac{N'_f}{2} + \tilde{N}'_c - N_c$ while the linking number of $NS5_R$ -brane from Figure 1 is $L_5 = \frac{N'_f}{2} + N''_c - N'_c$. Due to the connection of N''_c D4-branes with $NS5_R$ -brane

in Figure 1, the presence of N_c'' in the linking number arises. From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}'_c = N'_f + N_c'' + N_c - N'_c. \quad (2.4)$$

Compared with the previous case given by (2.3), the dependence on the ranks of both the first gauge group and the third gauge group occurs.

Let us draw this magnetic brane configuration in Figure 3A and recall that we put the coincident N'_f D6-branes in the nonzero v -direction in the electric theory as well as massless flavors for $Q(Q'')$ and $\tilde{Q}(\tilde{Q}'')$ for simplicity. If we ignore the $NS5_L$ -brane, N_c D4-branes, N_f D6-branes, N_f'' D6-branes, the $NS5'_R$ -brane and N_c'' D4-branes (detaching these branes from Figure 3A), then this brane configuration leads to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}'_c = N'_f - N'_c)$ with N'_f massive flavors [3, 6, 7].

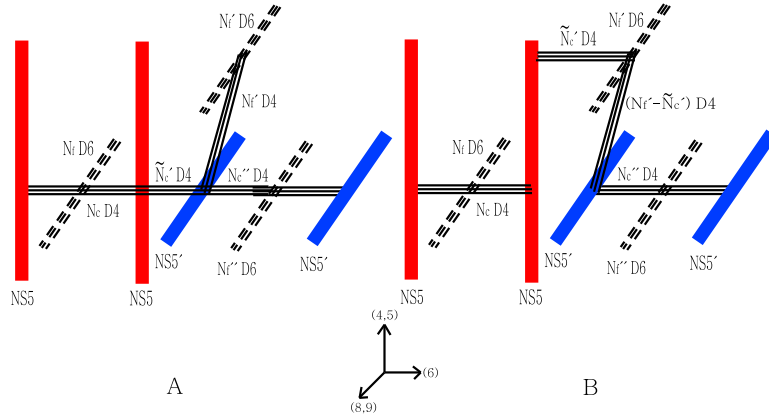


Figure 3: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $SU(N_c) \times SU(\tilde{N}'_c) \times SU(N_c'')$ gauge group with fundamentals $Q(q')[Q'']$ and $\tilde{Q}(\tilde{q}')[\tilde{Q}'']$ for each gauge group and bifundamentals $F(g)$ and $\tilde{F}(\tilde{g})$ and gauge singlets in Figure 3A. In Figure 3B, the non-supersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q(Q'')$ and $\tilde{Q}(\tilde{Q}'')$ is given.

In the dual theory, since there exist N'_f quarks q' , N'_f quarks \tilde{q}' , one bifundamental field g (which gives rise to the contribution of N_c'') and its complex conjugate field \tilde{g} (which gives the contribution of N_c''), one bifundamental field F (which will give rise to the contribution of N_c), and its complex conjugate field \tilde{F} (which will give rise to the contribution of N_c), the coefficient of the beta function for the second gauge group factor is $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N_c'' - N_c = 2N'_f + 2N_c'' + 2N_c - 3N'_c$ where we inserted the number of colors given in (2.4) in the second equality. Since there exist N_f'' quarks Q'' , N_f'' quarks \tilde{Q}'' , one bifundamental field g (which will give rise to the contribution of \tilde{N}'_c), its complex conjugate \tilde{g} (which will give rise to

the contribution of \tilde{N}'_c), N'_f -fields X'' , its complex conjugate N'_f -fields \tilde{X}'' , and the singlet Φ'' (which will give rise to N''_c), the coefficient of the beta function of second gauge group factor is $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - \tilde{N}'_c - N'_f - N''_c = N''_c + N'_c - N_c - 2N'_f - N''_f$. Similarly, since there exist N_f quarks Q , N_f quarks \tilde{Q} , one bifundamental field F (which will give rise to the contribution of \tilde{N}'_c), and its complex conjugate field \tilde{F} (which will give rise to the contribution of \tilde{N}'_c), the coefficient of the beta function of the first gauge group factor is $b_{SU(N_c)}^{mag} = 3N_c - N_f - \tilde{N}'_c = 2N_c - N''_c + N'_c - N_f - N'_f$.

Therefore, $SU(N_c)$, $SU(\tilde{N}'_c)$ and $SU(N''_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One relies on the perturbative calculations at low energy for this magnetic IR free region $b_{SU(N_c)}^{mag} < 0$, $b_{SU(\tilde{N}'_c)}^{mag} < 0$ and $b_{SU(N''_c)}^{mag} < 0$. The $SU(N_c)$ fields in the magnetic theory are different from those of electric theory and also the $SU(N''_c)$ fields in the magnetic theory are different from those of electric theory. Neglecting the $SU(N_c)$ and $SU(N''_c)$ dynamics, the magnetic $SU(\tilde{N}'_c)$ is IR free when

$$N'_c - N''_c - N_c < N'_f < \frac{3}{2}N'_c - N''_c - N_c.$$

Here the N_c - and N''_c -dependence appears.

Then one can summarize the gauge group and matter contents where there are additional fields X'' , \tilde{X}'' , M' and Φ'' as follows:

gauge group :	$SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$		
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$ $q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$ $Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$ $F \oplus \tilde{F}$ $g \oplus \tilde{g}$ $(X''_{n'} \equiv) \tilde{G}Q' \oplus G\tilde{Q}' (\equiv \tilde{X}''_{\tilde{n}'})$ $(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$ $(\Phi'' \equiv) G\tilde{G}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square})$ $(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$ $(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$ $(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square})$ $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ $(\mathbf{1}, \mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(f, \tilde{f} = 1, \dots, N_f)$ $(f', \tilde{f}' = 1, \dots, N'_f)$ $(f'', \tilde{f}'' = 1, \dots, N''_f)$ $(n', \tilde{n}' = 1, \dots, N'_f)$ $(f', \tilde{g}' = 1, \dots, N'_f)$

The dual magnetic superpotential, by adding the mass term (2.2) for Q' and \tilde{Q}' in the electric theory which is equal to put a linear term in M' in the dual magnetic theory, is given by

$$W_{dual} = \left(M' q' \tilde{q}' + g \tilde{X}'' \tilde{q}' + \tilde{g} q' X'' + \Phi'' g \tilde{g} \right) + m' M'.$$

One sees also the usual cubic interaction terms between the additional fields X'', \tilde{X}'', M' and Φ'' and their dual expressions, $\tilde{g}q', g\tilde{q}', q'\tilde{q}'$ and $g\tilde{g}$ respectively [14]. Here q' and \tilde{q}' are fundamental and antifundamental for the gauge group index respectively. Then, $q'\tilde{q}'$ has rank \tilde{N}'_c while m' has a rank N'_f . Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M' , cannot be satisfied if the rank N'_f exceeds \tilde{N}'_c . This is so-called rank condition and the supersymmetry is broken. Other F-term equations are satisfied by taking the vacuum expectation values of g, \tilde{g}, X'' and \tilde{X}'' to vanish.

Then the solutions for these equations can be written as follows:

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle g \rangle &= \langle \tilde{g} \rangle = \langle X'' \rangle = \langle \tilde{X}'' \rangle = 0. \end{aligned}$$

As we did in previous case, one can analyze the one loop computation by expanding the fields around the vacua and it will lead to the fact that states are stable by realizing the mass of $m_{M'_0}^2$ positive.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 3B. If we ignore the $NS5_L$ -brane, N_c D4-branes, N_f D6-branes, $NS5'_R$ -brane, N''_c D4-branes and N''_f D6-branes (detaching these from Figure 3B), as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}'_c = N'_f - N'_c)$ with N'_f massive flavors [3, 6, 7].

The nonsupersymmetric minimal energy brane configuration Figure 3B with vanishing N''_f D6-branes leads to the Figure 3 of [14] with a reflection with respect to the NS5-brane if we ignore the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes. Moreover, this Figure 3B with a replacement N'_f D6-branes by the NS5'-brane (and neglecting the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes) will become the Figure 2B of [19] with a rotation of $NS5_R$ -brane by $\frac{\pi}{2}$ angle or the Figure 4B of [19] with a reflection with respect to the $NS5_R$ -brane if we ignore the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes from the Figure 3B.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5_L$ -brane to the right, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 3 with a reflection with respect to the NS5-brane.

By lifting the type IIA description to M-theory, the corresponding magnetic M5-brane configuration, in a background space of $xt = v^{N_f + N''_f} \prod_{k=1}^{N'_f} (v - e_k)$ where e_k is the position of the N'_f D6-branes in the v direction, is described by

$$t^4 + (v^{N_c + N_f} + \dots)t^3 + (v^{\tilde{N}'_c + N_f} + \dots)t^2 + (v^{N''_c + 2N_f} + \dots)t + v^{3N_f + N''_f} (v - m)^{N'_f} = 0.$$

From this curve of quartic equation for t above, the asymptotic regions for four NS5-branes can be classified as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{N_c+N_f} + \dots & NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{\tilde{N}'_c-N_c} + \dots & NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{N''_c-\tilde{N}'_c+N_f} + \dots & NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{N_f+N'_f+N''_f-N''_c} + \dots & NS'_R \text{ asymptotic region.} \end{aligned}$$

2.4 Magnetic theory with dual for second gauge group

For the dualizing the middle gauge group, there exists another magnetic brane configuration. One considers dualizing one of the gauge groups regarding as the other gauge groups as a spectator, as we did in previous subsections. Also we consider the case where $\Lambda_2 \gg \Lambda_1, \Lambda_3$, in other words, the dualized group's dynamical scale is far above that of the other spectator groups.

Let us move the $NS5'_L$ -brane in Figure 1 to the right all the way past the $NS5_R$ -brane. After this brane motion, one arrives at the Figure 4A. That is, we rotate N'_f D6-branes a little bit (this does not change the classical electric superpotential as we explained before) and after dualizing the second gauge group, we rotate those N'_f D6-branes with opposite direction. The linking number of $NS5'_L$ -brane from Figure 4A is $L_5 = \frac{N'_f}{2} - \tilde{N}'_c + N''_c$. On the other hand, the linking number of $NS5'_L$ -brane from modified Figure 1 is $L_5 = -\frac{N'_f}{2} + N'_c - N_c$. Due to the connection of N_c D4-branes with $NS5'_L$ -brane in Figure 1, the presence of N_c in the linking number arises. From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}'_c = N'_f + N''_c + N_c - N'_c \quad (2.5)$$

which is the same as before given in (2.4).

Let us draw this magnetic brane configuration in Figure 4A and recall that we put the coincident N'_f D6-branes in the nonzero v -direction and the coincident N_f and N'_f D6-branes at $v = 0$ in the electric theory.

In the dual theory, since there exist N'_f quarks q' , N'_f quarks \tilde{q}' , one bifundamental field G (which will give rise to the contribution of N''_c), its complex conjugate field \tilde{G} (which will give rise to the contribution of N''_c), one bifundamental field f which (will give rise to the

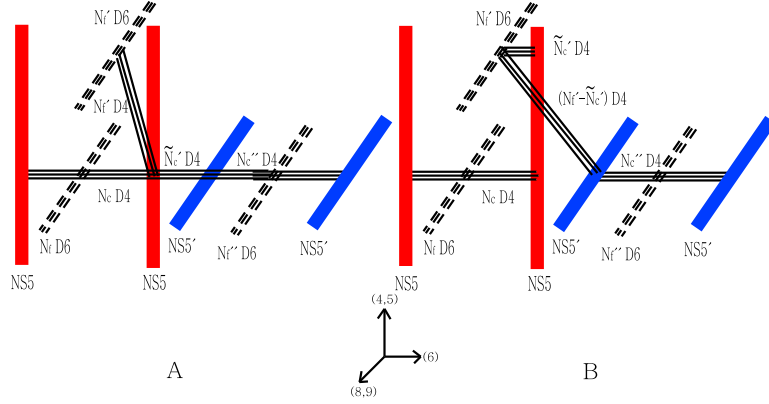


Figure 4: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$ gauge group with fundamentals $Q(q')[Q'']$ and $\tilde{Q}(\tilde{q}')[\tilde{Q}'']$ for each gauge group and bifundamentals $f(G)$ and $\tilde{f}(\tilde{G})$ and gauge singlets in Figure 4A. In Figure 4B, the non-supersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q(Q'')$ and $\tilde{Q}(\tilde{Q}'')$ is given.

contribution of N_c), and its complex conjugate field \tilde{f} (which will give rise to the contribution of N_c), the coefficient of the beta function for the second gauge group factor is $b_{SU(\tilde{N}'_c)}^{mag} = 3\tilde{N}'_c - N'_f - N''_c - N_c = 2N'_f + 2N''_c + 2N_c - 3N'_c$ where we inserted the number of colors given in (2.5) in the second equality and this is equal to the $b_{SU(\tilde{N}'_c)}^{mag}$ in previous subsection. Since there exist N'_f quarks Q'' , N'_f quarks \tilde{Q}'' , one bifundamental field G (which will give rise to the contribution of \tilde{N}'_c), and its complex conjugate field \tilde{G} (which will give rise to the contribution of \tilde{N}'_c), the coefficient of the beta function of second gauge group factor is $b_{SU(N''_c)}^{mag} = 3N''_c - N'_f - \tilde{N}'_c = 2N''_c + N'_c - N_c - N'_f - N''_f$. Similarly, since there exist N_f quarks Q , N_f quarks \tilde{Q} , one bifundamental field f (which will give rise to the contribution of \tilde{N}'_c), its complex conjugate field \tilde{f} (which will give rise to the contribution of \tilde{N}'_c), N'_f -fields X , its complex conjugate N'_f -fields \tilde{X} , and the singlet Φ (which will give rise to N_c), the coefficient of the beta function of the first gauge group factor is $b_{SU(N_c)}^{mag} = 3N_c - N_f - \tilde{N}'_c - N'_f - N_c = N_c - N''_c + N'_c - N_f - 2N'_f$.

Therefore, $SU(N_c)$, $SU(\tilde{N}'_c)$ and $SU(N''_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One relies on the perturbative calculations at low energy for this magnetic IR free region $b_{SU(N_c)}^{mag} < 0$, $b_{SU(\tilde{N}'_c)}^{mag} < 0$ and $b_{SU(N''_c)}^{mag} < 0$. Neglecting the $SU(N_c)$ and $SU(N''_c)$ dynamics, the magnetic $SU(\tilde{N}'_c)$ is IR free when

$$N'_c - N''_c - N_c < N'_f < \frac{3}{2}N'_c - N''_c - N_c.$$

Then one can summarize the gauge group and matter contents where there are additional

fields X, \tilde{X}, M' and Φ as follows:

gauge group :	$SU(N_c) \times SU(\tilde{N}'_c) \times SU(N''_c)$	
matter :	$Q_f \oplus \tilde{Q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (f, \tilde{f} = 1, \dots, N_f)$
	$q'_{f'} \oplus \tilde{q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad (f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square}) \quad (f'', \tilde{f}'' = 1, \dots, N''_f)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$
	$(X_{n'} \equiv) FQ' \oplus \tilde{F}\tilde{Q}' (\equiv \tilde{X}_{\tilde{n}'})$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1}) \quad (n', \tilde{n}' = 1, \dots, N'_f)$
	$(M'_{f', \tilde{g}'} \equiv) Q'\tilde{Q}'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad (f', \tilde{g}' = 1, \dots, N'_f)$
	$(\Phi \equiv) F\tilde{F}$	$(\mathbf{adj}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$

The dual magnetic superpotential, by adding the mass term (2.2) for Q' and \tilde{Q}' in the electric theory, which is equal to put a linear term in M' in the dual magnetic theory, is given by

$$W_{dual} = \left(M' q' \tilde{q}' + f X q' + \tilde{f} \tilde{q}' \tilde{X} + \Phi f \tilde{f} \right) + m' M'.$$

Here q' and \tilde{q}' are fundamental and antifundamental for the gauge group index respectively [14]. Then, $q'\tilde{q}'$ has rank \tilde{N}'_c while m' has a rank N'_f . Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M' , cannot be satisfied if the rank N'_f exceeds \tilde{N}'_c . This is so-called rank condition and the supersymmetry is broken. Other F-term equations are satisfied by taking the vacuum expectation values of f, \tilde{f}, X and \tilde{X} to vanish.

Then the solutions can be written as follows:

$$\begin{aligned} \langle q' \rangle &= \begin{pmatrix} \sqrt{m} e^\phi \mathbf{1}_{\tilde{N}'_c} \\ 0 \end{pmatrix}, \langle \tilde{q}' \rangle = \begin{pmatrix} \sqrt{m} e^{-\phi} \mathbf{1}_{\tilde{N}'_c} & 0 \end{pmatrix}, \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0 \mathbf{1}_{N'_f - \tilde{N}'_c} \end{pmatrix}, \\ \langle f \rangle &= \langle \tilde{f} \rangle = \langle X \rangle = \langle \tilde{X} \rangle = 0. \end{aligned}$$

As we did in previous case, one can analyze the one loop computation by expanding the fields around the vacua and it will lead to the fact that states are stable by realizing the mass of $m_{M'_0}^2$ positive.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 4B. If we ignore the $NS5_L$ -brane, N_c D4-branes, N_f D6-branes, $NS5'_R$ -brane, N''_c D4-branes and N''_f D6-branes (detaching these from Figure 4B), then this brane configuration looks similar to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD

with the magnetic gauge group $SU(\tilde{N}'_c = N'_f - N'_c)$ with N'_f massive flavors. The difference occurs in the position of D6-branes.

The nonsupersymmetric minimal energy brane configuration Figure 4B with a replacement N'_f D6-branes by the NS5'-brane (neglecting the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes with vanishing N_f D6-branes) leads to the Figure 5B of [19] with a rotation of $NS5'_L$ -brane by $\frac{\pi}{2}$ angle.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5'_R$ -brane to the left, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 4 with a reflection with respect to the NS5-brane.

After lifting the type IIA description to M-theory, the magnetic M5-brane configuration, in a background space of $xt = v^{N_f+N''_f} \prod_{k=1}^{N'_f} (v - e_k)$, is described by

$$t^4 + (v^{N_c} + \dots)t^3 + \left[v^{\tilde{N}'_c+N_f}(v-m)^{N'_f} + \dots \right] t^2 + \left[v^{N''_c+2N_f}(v-m)^{2N'_f} + \dots \right] t + v^{3N_f+N''_f}(v-m)^{3N'_f} = 0.$$

From this curve of quartic equation for t above, the asymptotic regions for four NS5-branes can be classified as follows

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{N_c} + \dots \quad NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{\tilde{N}'_c-N_c+N_f+N'_f} + \dots \quad NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{N''_c-\tilde{N}'_c+N_f+N'_f} + \dots \quad NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{N_f+N'_f+N''_f-N''_c} + \dots \quad NS'_R \text{ asymptotic region.} \end{aligned}$$

2.5 Magnetic theory with dual for first gauge group

Now we turn to the last magnetic brane configuration. One considers dualizing one of the gauge groups regarding as the other gauge groups as a spectator, as done in previous subsections. Also we consider the case where $\Lambda_1 \gg \Lambda_2, \Lambda_3$, in other words, the dualized group's dynamical scale is far above that of the other spectator groups.

Let us move the $NS5_L$ -brane in Figure 1 to the right all the way past the $NS5'_L$ -brane. After this brane motion, one arrives at the Figure 5A. Recall that the N_f D6-branes are perpendicular to the $NS5_L$ -brane in Figure 1. The linking number of $NS5_L$ -brane from Figure 5A is $L_5 = \frac{N_f}{2} - \tilde{N}_c + N'_c$. Due to the connection of N'_c D4-branes with $NS5_L$ -brane

in Figure 5A, the presence of N'_c in the linking number arises. On the other hand, the linking number of $NS5_L$ -brane from Figure 1 is $L_5 = -\frac{N_f}{2} + N_c$. From these two relations, one obtains the number of colors of dual magnetic theory

$$\tilde{N}_c = N_f + N'_c - N_c. \quad (2.6)$$

Let us draw this magnetic brane configuration in Figure 5A and recall that we put the coincident N_f D6-branes in the nonzero v -direction as well as N'_f and N''_f D6-branes at $v = 0$ in the electric theory. If we ignore the $NS5_R$ -brane, N'_c D4-branes, N'_f D6-branes, the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes(detaching these branes from Figure 5A), then this brane configuration leads to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}_c = N_f - N_c)$ with N_f massive flavors [3, 6, 7].

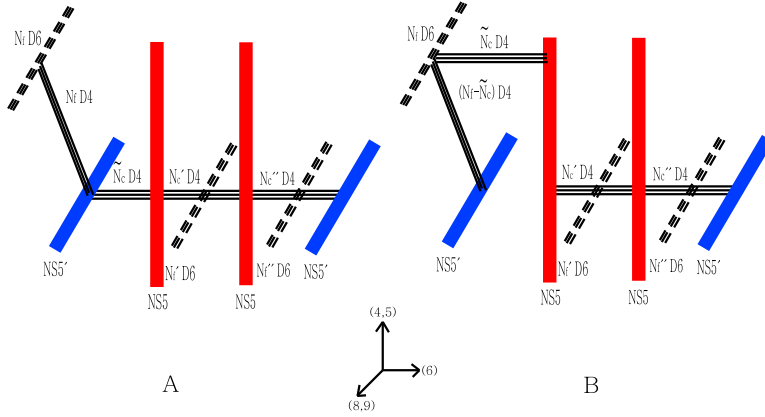


Figure 5: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$ gauge group with fundamentals $q(Q')[Q'']$ and $\tilde{q}(\tilde{Q}')[\tilde{Q}'']$ for each gauge group and bifundamentals $F(g)$ and $\tilde{F}(\tilde{g})$ and gauge singlets in Figure 5A. In Figure 5B, the non-supersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q'(Q'')$ and $\tilde{Q}'(\tilde{Q}'')$ is given.

In the dual theory, since there exist N_f quarks q , N_f quarks \tilde{q} , one bifundamental field f (which will give rise to the contribution of N'_c), and its complex conjugate \tilde{f} (which will give rise to the contribution of N'_c), the coefficient of the beta function for the third gauge group factor is $b_{SU(\tilde{N}_c)}^{mag} = 3\tilde{N}_c - N_f - N'_c = 2N_f + 2N'_c - 3N_c$ where we inserted the number of colors given in (2.6) in the second equality. Since there exist N'_f quarks Q' , N'_f quarks \tilde{Q}' , one bifundamental field f (which will give rise to the contribution of \tilde{N}_c), its complex conjugate field \tilde{f} (which will give rise to the contribution of \tilde{N}_c), one bifundamental field G (which will give rise to the contribution of N''_c), its complex conjugate \tilde{G} (which will give rise to the contribution of N''_c), N_f fields X' , its complex conjugate N_f fields \tilde{X}' , and the

singlet Φ' (which will give rise to N'_c), the coefficient of the beta function of second gauge group factor is $b_{SU(N'_c)}^{mag} = 3N'_c - N'_f - \tilde{N}_c - N''_c - N_f - N'_c = N'_c + N_c - N''_c - 2N_f - N'_f$. Similarly, since there exist N''_f quarks Q'' , N''_f quarks \tilde{Q}'' , one bifundamental field G (which will give rise to the contribution of N'_c), and its complex conjugate field \tilde{G} (which will give rise to the contribution of N'_c), the coefficient of the beta function of the third gauge group factor is $b_{SU(N''_c)}^{mag} = 3N''_c - N''_f - N'_c = b_{SU(N''_c)}$.

Therefore, $SU(\tilde{N}_c)$, $SU(N'_c)$ and $SU(N''_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One relies on the perturbative calculations at low energy for this magnetic IR free region $b_{SU(\tilde{N}_c)}^{mag} < 0$, $b_{SU(N_c)}^{mag} < 0$ and $b_{SU(N''_c)}^{mag} < 0$. Note that the $SU(N'_c)$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SU(N'_c)} - b_{SU(N'_c)}^{mag} > 0$, $SU(N'_c)$ is more asymptotically free than $SU(N'_c)^{mag}$. Neglecting the $SU(N'_c)$ and $SU(N''_c)$ dynamics, the magnetic $SU(\tilde{N}_c)$ is IR free when

$$N_c - N'_c < N_f < \frac{3}{2}N_c - N'_c.$$

Then one can summarize the gauge group and matter contents where there are additional fields X' , \tilde{X}' , M and Φ' as follows:

gauge group :	$SU(\tilde{N}_c) \times SU(N'_c) \times SU(N''_c)$		
matter :	$q_f \oplus \tilde{q}_{\tilde{f}}$	$(\square, \mathbf{1}, \mathbf{1}) \oplus (\overline{\square}, \mathbf{1}, \mathbf{1})$	$(f, \tilde{f} = 1, \dots, N_f)$
	$Q'_{f'} \oplus \tilde{Q}'_{\tilde{f}'}$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1})$	$(f', \tilde{f}' = 1, \dots, N'_f)$
	$Q''_{f''} \oplus \tilde{Q}''_{\tilde{f}''}$	$(\mathbf{1}, \mathbf{1}, \square) \oplus (\mathbf{1}, \mathbf{1}, \overline{\square})$	$(f'', \tilde{f}'' = 1, \dots, N''_f)$
	$f \oplus \tilde{f}$	$(\square, \overline{\square}, \mathbf{1}) \oplus (\overline{\square}, \square, \mathbf{1})$	
	$G \oplus \tilde{G}$	$(\mathbf{1}, \square, \overline{\square}) \oplus (\mathbf{1}, \overline{\square}, \square)$	
	$(X'_n \equiv) \tilde{F}Q \oplus F\tilde{Q} (\equiv \tilde{X}'_n)$	$(\mathbf{1}, \square, \mathbf{1}) \oplus (\mathbf{1}, \overline{\square}, \mathbf{1})$	$(n, \tilde{n} = 1, \dots, N_f)$
	$(M_{f, \tilde{g}} \equiv) Q\tilde{Q}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(f, \tilde{g} = 1, \dots, N_f)$
	$(\Phi' \equiv) F\tilde{F}$	$(\mathbf{1}, \mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$	

The dual magnetic superpotential, by adding the mass term (2.2) for Q and \tilde{Q} in the electric theory, which is equal to put a linear term in M in the dual magnetic theory, is given by [14]

$$W_{dual} = \left(Mq\tilde{q} + f\tilde{X}'\tilde{q} + \tilde{f}qX' + \Phi'f\tilde{f} \right) + mM.$$

Here q and \tilde{q} are fundamental and antifundamental for the gauge group index respectively. Then, $q\tilde{q}$ has rank \tilde{N}_c while m has a rank N_f . Therefore, the F-term condition, the derivative

the superpotential W_{dual} with respect to M , cannot be satisfied if the rank N_f exceeds \tilde{N}_c . This is so-called rank condition and the supersymmetry is broken. Other F-term equations are satisfied by taking the vacuum expectation values of f, \tilde{f}, X' and \tilde{X}' to vanish.

Then the solutions can be written as follows:

$$\begin{aligned} \langle q \rangle &= \begin{pmatrix} \sqrt{m}e^\phi \mathbf{1}_{\tilde{N}_c} \\ 0 \end{pmatrix}, \langle \tilde{q} \rangle = \begin{pmatrix} \sqrt{m}e^{-\phi} \mathbf{1}_{\tilde{N}_c} & 0 \end{pmatrix}, \langle M \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{pmatrix}, \\ \langle f \rangle &= \langle \tilde{f} \rangle = \langle X' \rangle = \langle \tilde{X}' \rangle = 0. \end{aligned}$$

As we did in previous case, one can analyze the one loop computation by expanding the fields around the vacua and it will lead to the fact that states are stable by realizing the mass of $m_{M_0}^2$ positive.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 5B. If we ignore the $NS5_R$ -brane, N'_c D4-branes, N'_f D6-branes, $NS5'_R$ -brane, N''_c D4-branes and N''_f D6-branes (detaching these from Figure 5B), as observed already, then this brane configuration is the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}_c = N_f - N_c)$ with N_f massive flavors [3, 6, 7].

The nonsupersymmetric minimal energy brane configuration Figure 5B with a replacement N_f D6-branes by the $NS5'$ -brane (neglecting the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes and N'_f D6-branes) leads to the Figure 4B of [19].

When we move the $NS5'_L$ -brane in Figure 1 to the left all the way past the $NS5_L$ -brane, then one arrives at the magnetic brane configuration similar to the Figure 5. The only difference is that the N_f D6-branes are located at the right hand side of the $NS5_L$ -brane. Then this nonsupersymmetric minimal energy brane configuration with a replacement N_f D6-branes by the $NS5'$ -brane (by neglecting the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes) leads to the Figure 5B of [19] with a reflection with respect to the $NS5$ -brane and an extra rotation of $NS5'_L$ -brane by $\frac{\pi}{2}$ angle.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5_L$ -brane to the left, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 2 with a reflection with respect to the $NS5$ -brane. Furthermore, by moving the $NS5'_R$ -brane to the right, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the new figure in previous paragraph with a reflection with respect to the $NS5_L$ -brane.

The corresponding magnetic M5-brane configuration, in a background space of $xt = v^{N'_f + N''_f} \prod_{k=1}^{N_f} (v - e_k)$ where this four dimensional space replaces (45610) directions, is de-

scribed by

$$t^4 + (v^{\tilde{N}_c} + \dots)t^3 + \left[v^{N'_c}(v-m)^{N_f} + \dots \right] t^2 + \left[v^{N''_c+N'_f}(v-m)^{2N_f} + \dots \right] t + (v-m)^{3N_f} v^{2N'_f+N''_f} = 0.$$

From this curve of quartic equation for t above, the asymptotic regions for four NS5-branes can be classified as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{N'_c - \tilde{N}_c + N_f} + \dots \quad NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{N_f + N'_f + N''_c - N'_c} + \dots \quad NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{\tilde{N}_c} + \dots \quad NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{N_f + N'_f + N''_f - N''_c} + \dots \quad NS'_R \text{ asymptotic region.} \end{aligned}$$

2.6 Magnetic theories for the multiple product gauge groups

Now one can generalize the method for the triple product gauge groups to the finite n -multiple product gauge groups characterized by [46, 47]

$$SU(N_{c,1}) \times \dots \times SU(N_{c,n})$$

with the matter, the $(n-1)$ bifundamentals $(\square_1, \bar{\square}_2, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \square_{n-1}, \bar{\square}_n)$, their complex conjugate $(\bar{\square}_1, \square_2, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \bar{\square}_{n-1}, \square_n)$, linking the gauge groups together, n -fundamentals $(\square_1, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \square_n)$, and n -antifundamentals $(\bar{\square}_1, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \bar{\square}_n)$. Then the mass-deformed superpotential can be written as $W_{elec} = \sum_{i=1}^n m_i Q_i \tilde{Q}_i$. The brane configuration can be constructed similarly and any two neighboring NS-branes are perpendicular to each other.

There exist $(2n-2)$ magnetic theories and they can be classified as four cases as follows.

- Case 1

When the Seiberg dual is taken for the first gauge group factor by assuming that $\Lambda_1 \gg \Lambda_i$ where $i = 2, \dots, n$, one follows the procedure given in the subsection 2.5. The gauge group is

$$SU(\tilde{N}_{c,1} \equiv N_{f,1} + N_{c,2} - N_{c,1}) \times SU(N_{c,2}) \times \dots \times SU(N_{c,n})$$

and the matter contents are given by the dual quarks q_1 $(\square_1, \mathbf{1}, \dots, \mathbf{1})$ and \tilde{q}_1 in the representation $(\bar{\square}_1, \mathbf{1}, \dots, \mathbf{1})$ as well as $(n-1)$ quarks Q_i and \tilde{Q}_i where $i = 2, \dots, n$, the bifundamentals f_1 in the representation $(\square_1, \bar{\square}_2, \mathbf{1}, \dots, \mathbf{1})$ under the dual gauge group, \dots ,

and \tilde{f}_1 in the representation $(\overline{\square}_1, \square_2, \mathbf{1}, \dots, \mathbf{1})$ under the dual gauge group in addition to $(n-2)$ bifundamentals G_i and \tilde{G}_i , and various gauge singlets X_2, \tilde{X}_2, M_1 and Φ_2 . The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 5. The magnetic superpotential can be written as $W_{dual} = \left(M_1 q_1 \tilde{q}_1 + f_1 \tilde{X}_2 \tilde{q}_1 + \tilde{f}_1 q_1 X_2 + \Phi_2 f_1 \tilde{f}_1 \right) + m_1 M_1$. By computing the contribution for the one loop as in the subsection 2.5, the vacua are stable and the asymptotic behavior of $(n+1)$ NS-branes can be obtained also.

• Case 2

When the Seiberg dual is taken for the last gauge group factor by assuming that $\Lambda_n \gg \Lambda_i$ where $i = 1, 2, \dots, (n-1)$, one follows the procedure given in the subsection 2.2. The gauge group is given by

$$SU(N_{c,1}) \times \dots \times SU(N_{c,n-1}) \times SU(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n}).$$

The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane of Figure 2. The magnetic superpotential can be written as $W_{dual} = \left(M_n q_n \tilde{q}_n + g_{n-1} X_{n-1} q_n + \tilde{g}_{n-1} \tilde{q}_n \tilde{X}_{n-1} + \Phi_{n-1} g_{n-1} \tilde{g}_{n-1} \right) + m_n M_n$.

• Case 3

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 2.3. The gauge group is given by

$$\dots \times SU(N_{c,i-1}) \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times SU(N_{c,i+1}) \times \dots$$

where $2 \leq i \leq n-1$ implying that the number of possible magnetic gauge group is $(n-2)$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 3. The magnetic superpotential can be written as $W_{dual} = \left(M_i q_i \tilde{q}_i + g_i \tilde{X}_{i+1} \tilde{q}_i + \tilde{g}_i q_i X_{i+1} + \Phi_{i+1} g_i \tilde{g}_{i+1} \right) + m_i M_i$.

• Case 4

When the Seiberg dual is taken for the middle gauge group factor with different brane motion by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 2.4. The gauge group is given by

$$\dots \times SU(N_{c,i-1}) \times SU(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i}) \times SU(N_{c,i+1}) \times \dots$$

where $2 \leq i \leq n - 1$ implying that the number of possible magnetic gauge group is given by $(n - 2)$. The corresponding brane configuration can be obtained similarly and the extra $(i - 2)$ NS-branes, $(i - 2)$ sets of D6-branes and $(i - 2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n - i - 1)$ NS-branes, $(n - i - 1)$ sets of D6-branes and $(n - i - 1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 4. The magnetic superpotential can be written as $W_{dual} = \left(M_i q_i \tilde{q}_i + f_{i-1} X_{i-1} q_i + \tilde{f}_{i-1} \tilde{q}_i \tilde{X}_{i-1} + \Phi_{i-1} f_{i-1} \tilde{f}_{i-1} \right) + m_i M_i$.

3 Nonsupersymmetric meta-stable brane configurations of $Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$ and its multiple product gauge theories

3.1 Electric theory

The gauge group and matter contents are summarized as follows:

	gauge group :	$Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$	
matter :	Q_f	$(\square, \mathbf{1}, \mathbf{1})$	$(f = 1, \dots, 2N_f)$
	$Q'_{f'}$	$(\mathbf{1}, \square, \mathbf{1})$	$(f' = 1, \dots, 2N'_f)$
	$Q''_{f''}$	$(\mathbf{1}, \mathbf{1}, \square)$	$(f'' = 1, \dots, 2N''_f)$
	F	$(\square, \square, \mathbf{1})$	
	G	$(\mathbf{1}, \square, \square)$	

In the electric theory ⁴, since there exist $2N_f$ quarks Q , and one bifundamental field F (which will give rise to the contribution of $2N'_c$), the coefficient of the beta function of the first gauge group factor is $b_{Sp(N_c)} = 3(2N_c + 2) - 2N_f - 2N'_c$. Similarly, since there exist $2N'_f$ quarks Q' , one bifundamental field F (which will give rise to the contribution of $2N_c$), and one bifundamental field G (which will give rise to the contribution of $2N''_c$), the coefficient of the beta function of the second gauge group factor is $b_{SO(2N'_c)} = 3(2N'_c - 2) - 2N'_f - 2N_c - 2N''_c$. Finally, since there exist $2N''_f$ quarks Q'' , and one bifundamental field G (which will give rise to the contribution of $2N'_c$), the coefficient of the beta function of the third gauge group factor is $b_{Sp(N''_c)} = 3(2N''_c + 2) - 2N''_f - 2N'_c$.

The anomaly free global symmetry contains $SU(2N_f) \times SU(2N'_f) \times SU(2N''_f) \times U(1)_R$ and let us denote the strong coupling scales for $Sp(N_c)$ as Λ_1 , for $SO(2N'_c)$ as Λ_2 and for $Sp(N''_c)$

⁴For $Sp(N_c) \times SO(2N'_c + 1) \times Sp(N''_c)$ gauge theory, the analysis can be done similarly and we do not present here.

as Λ_3 . The theory is asymptotically free when $b_{Sp(N_c)} > 0$ for the $Sp(N_c)$ gauge theory, when $b_{SO(2N'_c)} > 0$ for the $SO(2N'_c)$ gauge theory, and when $b_{Sp(N''_c)} > 0$ for the $Sp(N''_c)$ gauge theory.

The classical electric superpotential can be obtained from (2.1) by orientifolding

$$W_{elec} = (\mu A^2 + \lambda Q A Q + F A F + \mu' A'^2 + \lambda' Q' A' Q' + F A' F + G A' G + \mu'' A''^2 + \lambda'' Q'' A'' Q'' + G A'' G) + m Q Q + m' Q' Q' + m'' Q'' Q'' \quad (3.1)$$

where the coefficient functions $\mu, \mu', \mu'', \lambda, \lambda'$ and λ'' are given by six rotation angles. Here the adjoint field for $Sp(N_c)$ gauge group is denoted by A , the adjoint field for $SO(2N'_c)$ gauge group is denoted by A' , and the adjoint field for $Sp(N''_c)$ gauge group is denoted by A'' . The mass terms of these adjoint fields are related to the rotation angles of NS-branes in type IIA brane configuration. The couplings of flavors with these adjoint fields are related also to the rotation angles of NS-branes as well as the rotation angles of D6-branes. We add the mass terms for each flavor. Setting the fields Q'', G , and A'' to zero, the superpotential becomes the one described in [15].

After integrating out the adjoint fields A, A' and A'' , this superpotential (3.1) at the particular orientations for branes, i.e., the case where any two neighboring NS-branes are perpendicular to each other, will reduce to the last three mass-deformed terms since the coefficient functions $\frac{1}{\mu}, \frac{1}{\mu'}$ and $\frac{1}{\mu''}$ vanish at this particular rotation angles for branes. Then the classical superpotential by deforming this theory by adding the mass terms for the quarks $Q(Q')[Q'']$ is given by

$$W_{elec} = m Q Q + m' Q' Q' + m'' Q'' Q''. \quad (3.2)$$

The type IIA brane configuration for this mass-deformed theory can be described by as follows. The $2N_c$ -color D4-branes (01236) are suspended between the $NS5_L$ -brane (012345) and the $NS5'_L$ -brane (012389) along x^6 direction, together with N_f D6-branes (0123789) which are parallel to $NS5'_L$ -brane and have nonzero $+v$ direction (and their mirrors). The $NS5_R$ -brane is located at the right hand side of the $NS5'_L$ -brane along the x^6 direction and there exist $2N'_c$ -color D4-branes suspended between them, with N'_f D6-branes which have nonzero $+v$ direction (and their mirrors). Moreover, the $NS5'_R$ -brane is located at the right hand side of the $NS5_R$ -brane along the x^6 direction and there exist $2N''_c$ -color D4-branes suspended between them, with N''_f D6-branes which have nonzero $+v$ direction (and their mirrors).

One summarizes the brane configuration as follows:

- $NS5_L(NS5_R)$ -brane in (012345) directions.
- $NS5'_L(NS5'_R)$ -brane in (012389) directions.

- $2N_c(2N'_c)[2N''_c]$ -color D4-branes in (01236) directions.
- $2N_f(2N'_f)[2N''_f]$ D6-branes in (0123789) directions.
- $O4^\pm$ -planes in (01236) directions

Now we draw this electric brane configuration in Figure 6 and we put the coincident $N_f(N'_f)[N''_f]$ D6-branes in the nonzero $+v$ direction (and their mirrors). The quarks $Q(Q')[Q'']$ correspond to strings between the $2N_c(2N'_c)[2N''_c]$ -color D4-branes with $2N_f(2N'_f)[2N''_f]$ D6-branes. The bifundamentals $F(G)$ correspond to strings stretching between the $2N_c(2N'_c)$ -color D4-branes with $2N'_c(2N''_c)$ -color D4-branes.

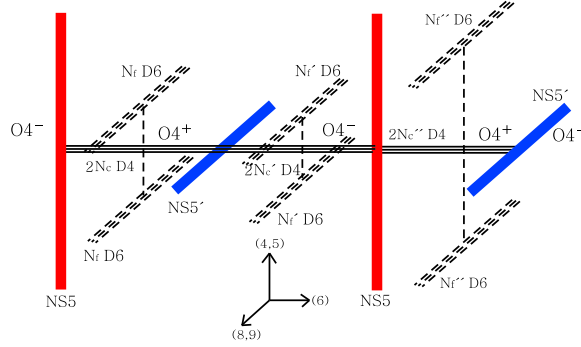


Figure 6: The $\mathcal{N} = 1$ supersymmetric electric brane configuration with $Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$ gauge group with flavors $Q(Q')[Q'']$ for each gauge group and bifundamentals $F(G)$.

3.2 Magnetic theory with dual for third gauge group

Let us take the Seiberg dual for the third gauge group factor $Sp(N''_c)$ while keeping the first and the second gauge group factors $Sp(N_c)$ and $SO(2N'_c)$ untouched. Suppose that $\Lambda_3 \gg \Lambda_1, \Lambda_2$. This can be done by type IIA string theory side via brane motion. After we move the $NS5_R$ -brane in Figure 6 to the right all the way past the $NS5'_R$ -brane, we arrives at the Figure 7A. Then the linking number of $NS5_R$ -brane from Figure 7A is given by $L_5 = \frac{(2N''_f)}{2} - 1 - (1) - 2\tilde{N}''_c$. Note that $O4^+$ -plane with RR charge +1 realizes a symplectic gauge group while $O4^-$ -plane with RR charge -1 does an orthogonal gauge group. Originally, it was $L_5 = -\frac{(2N''_f)}{2} + 1 - (-1) + 2N''_c - 2N'_c$ from Figure 6 before the brane motion. Therefore, by the linking number conservation and equating these two L_5 's each other, we are left with the number of colors in the magnetic theory $\tilde{N}''_c = N''_f + N'_c - N''_c - 2$. Note the dependence on N'_c here and the constant piece -2 comes from the presence of $O4$ -plane.

Let us draw this magnetic brane configuration in Figure 7A and we put the coincident N''_f D6-branes in the nonzero $+v$ direction (and its mirrors) as well as the massless $Q(Q')$. If we ignore the $NS5_L$ -brane, $2N_c$ D4-branes, $2N_f$ D6-branes, the $NS5'_L$ -brane, $2N'_c$ D4-branes and

$2N'_f$ D6-branes(detaching these branes from Figure 7A), then this brane configuration leads to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $Sp(\tilde{N}''_c = N''_f - N'_c - 2)$ with N''_f massive flavors [6, 11].

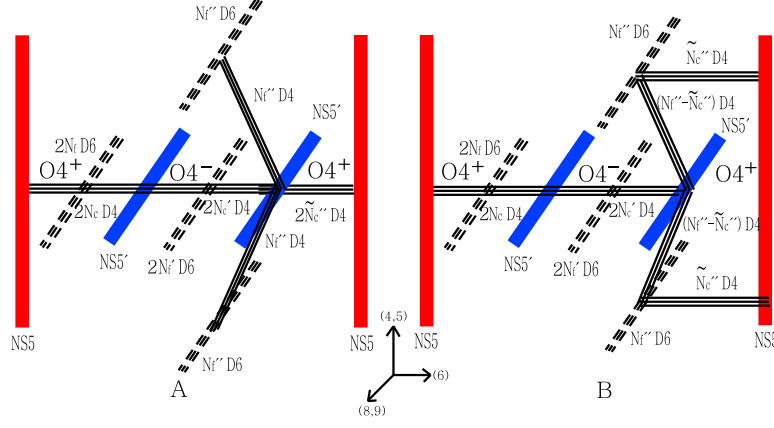


Figure 7: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $Sp(N_c) \times SO(2N'_c) \times Sp(\tilde{N}''_c)$ gauge group with flavors $Q(Q')[q'']$ for each gauge group, the bifundamentals $F(g)$, and gauge singlets in Figure 7A. In Figure 7B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q(Q')$ is given.

In the dual theory, since there exist $2N''_f$ fundamental fields q'' , and one bifundamental field g (which will give rise to the contribution of $2N'_c$), the coefficient of the beta function is $b_{Sp(\tilde{N}''_c)}^{mag} = 3(2\tilde{N}''_c + 2) - 2N''_f - 2N'_c$ and since there are $2N'_f$ fundamental fields Q' , $2\tilde{N}''_c$ fundamental fields g for the second factor, $2N_c$ fundamental fields F for the first factor, an antisymmetric tensor Φ' (which will contribute to $(2N'_c - 2)$), and $2N''_f$ fields X' , the coefficient of the beta function is $b_{SO(2N'_c)}^{mag} = 3(2N'_c - 2) - 2N'_f - 2\tilde{N}''_c - 2N_c - (2N'_c - 2) - 2N''_f$. Since there exist $2N_f$ quarks Q , and one bifundamental field F (which will give rise to the contribution of $2N'_c$), the coefficient of the beta function of the first gauge group factor is $b_{Sp(N_c)}^{mag} = 3(2N_c + 2) - 2N_f - 2N'_c = b_{Sp(N_c)}$. Note that the $Sp(N_c)$ fields in the magnetic theory are the same as those of the electric theory.

Therefore, both $Sp(\tilde{N}''_c)$, $SO(2N'_c)$, and $Sp(N_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b_{Sp(\tilde{N}''_c)}^{mag} < 0$, $b_{SO(2N'_c)}^{mag} < 0$, and $b_{Sp(N_c)}^{mag} < 0$. Note that the $SO(2N'_c)$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SO(2N'_c)} - b_{SO(2N'_c)}^{mag} > 0$, $SO(2N'_c)$ is more asymptotically free than $SO(2N'_c)^{mag}$. Neglecting the $SO(2N'_c)$ and $Sp(N_c)$ dynamics, the magnetic $Sp(\tilde{N}''_c)$ is IR free when

$$N''_c + 2 - N'_c < N''_f < \frac{3}{2}(N''_c + 1) - N'_c.$$

Then one can summarize the gauge group and matter contents where there are additional fields X' , M'' and Φ' as follows:

	gauge group :	$Sp(N_c) \times SO(2N'_c) \times Sp(\tilde{N}''_c)$	
matter :	Q_f	$(\square, \mathbf{1}, \mathbf{1})$	$(f = 1, \dots, 2N_f)$
	$Q'_{f'}$	$(\mathbf{1}, \square, \mathbf{1})$	$(f' = 1, \dots, 2N'_f)$
	$q''_{f''}$	$(\mathbf{1}, \mathbf{1}, \square)$	$(f'' = 1, \dots, 2N''_f)$
	F	$(\square, \square, \mathbf{1})$	
	g	$(\mathbf{1}, \square, \square)$	
	$(X'_{n''} \equiv) GQ''$	$(\mathbf{1}, \square, \mathbf{1})$	$(n'' = 1, \dots, 2N''_f)$
	$(M''_{f'', g''} \equiv) Q''Q''$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(f'', g'' = 1, \dots, 2N''_f)$
	$(\Phi' \equiv) GG$	$(\mathbf{1}, \text{asymm}, \mathbf{1})$	

The dual magnetic tree level superpotential, by adding the mass term for the Q'' in electric theory corresponding to add a linear term in M'' in dual magnetic theory, is given by

$$W_{dual} = (M'' q'' q'' + g X' q'' + \Phi' g g) + m'' M''.$$

Here q'' is fundamental for the gauge group index [15]. Then, $q'' q''$ has rank $2\tilde{N}''_c$ while m'' has a rank $2N''_f$. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M'' , cannot be satisfied if the $2N''_f$ exceeds $2\tilde{N}''_c$. This is so-called rank condition and the supersymmetry is broken.

More explicitly, the classical moduli space of vacua can be obtained from F-term equations. From the F-terms $F_{q''}$ and $F_{M''}$, one gets $M'' q'' + g X' = 0 = q'' q'' + m$. Similarly, one obtains $\Phi' g + X' q'' = 0 = g g$ from the F-terms F_g and $F_{\Phi'}$. Moreover, there is a relation $q'' g = 0$ from the F-term $F_{X'}$. Then, one obtains the following solutions

$$\langle q'' \rangle = \begin{pmatrix} i\sqrt{m} \mathbf{1}_{2\tilde{N}''_c} \\ 0 \end{pmatrix}, \langle M'' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M''_0 \mathbf{1}_{(N''_f - \tilde{N}''_c)} \otimes i\sigma^2 \end{pmatrix}, \langle g \rangle = 0 = \langle X' \rangle$$

where $M''_0 \mathbf{1}_{(N''_f - \tilde{N}''_c)} \otimes i\sigma^2$ is an arbitrary $2(N''_f - \tilde{N}''_c) \times 2(N''_f - \tilde{N}''_c)$ antisymmetric matrix and the zeros of $\langle q'' \rangle$ are $2(N''_f - \tilde{N}''_c) \times 2\tilde{N}''_c$ zero matrices. Similarly, the zeros of $2N''_f \times 2N''_f$ matrix M'' are assumed also.

Then the superpotential becomes

$$W_{dual}^{fluct} = M''_0 (\delta\varphi \delta\varphi + m) + \delta Z^T \delta\varphi i\sqrt{m} + \delta Z i\sqrt{m} \delta\varphi$$

by following the fluctuation for the fields

$$q'' = \begin{pmatrix} i\sqrt{m}\mathbf{1}_{2\tilde{N}_c''} + (\delta\chi_A + \delta\chi_S)\mathbf{1}_{\tilde{N}_c''} \otimes i\sigma^2 \\ \delta\varphi \end{pmatrix}, \quad M'' = \begin{pmatrix} \delta Y & \delta Z^T \\ -\delta Z & M_0''\mathbf{1}_{(N_f''-\tilde{N}_c'')} \otimes i\sigma^2 \end{pmatrix}$$

as well as the fluctuations for g and X' . There are two kinds of terms, the vacuum of $\langle q'' \rangle$ multiplied by $\delta X' \delta g$ and the vacuum of $\langle \Phi' \rangle$ multiplied by $\delta g \delta g$. By redefining these as $\delta \hat{X}' \delta \hat{g}$ and $\delta \hat{g} \delta \hat{g}$ respectively, they do not enter the contributions for the one loop result. At one loop, the effective potential $V_{eff}^{(1)}$ for M_0'' can be obtained from this superpotential which consists of the matrices M and N of [52] where the defining function $\mathcal{F}(v^2)$ can be computed. Using the equation (2.14) of [52] of $m_{M_0''}^2$ and $\mathcal{F}(v^2)$, one gets that $m_{M_0''}^2$ will contain $(\log 4 - 1) > 0$. This implies that these vacua are stable.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 7B. If we ignore the $NS5_L$ -brane, $2N_c$ D4-branes, $2N_f$ D6-branes, the $NS5'_L$ -brane, $2N'_c$ D4-branes and $2N'_f$ D6-branes (detaching these branes from Figure 7B), then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $Sp(\tilde{N}_c'')$ with N_f'' massive flavors [6, 11].

The nonsupersymmetric minimal energy brane configuration Figure 7B with vanishing $2N'_f$ D6-branes leads to the Figure 3 of [15] if we ignore the $NS5_L$ -brane, $2N_f$ D6-branes and $2N_c$ D4-branes. Moreover, this Figure 7B with a replacement N_f'' D6-branes by the NS5'-brane (neglecting the $NS5_L$ -brane, $2N_f$ D6-branes and $2N_c$ D4-branes) will become the Figure 7B of [19] with a reflection with respect to the $NS5_L$ -brane and a rotation of $NS5_R$ -brane by $\frac{\pi}{2}$ angle.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5_R$ -brane to the left, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 10B (which will appear later) with a reflection with respect to the NS5-brane. Furthermore, by moving the $NS5'_L$ -brane to the right, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the new figure in previous paragraph with a reflection with respect to the $NS5_L$ -brane.

After lifting the type IIA description we explained so far to M-theory, the corresponding magnetic M5-brane configuration, in a background space of $xt = v^{2N_f+2N'_f} \prod_{k=1}^{N_f''} (v^2 - e_k^2)$ where this four dimensional space replaces (45610) directions, is characterized by [55]

$$t^4 + (v^{2N_c+2} + \dots)t^3 + (v^{2N'_c+2N_f} + \dots)t^2 \\ + (v^{2\tilde{N}_c''+2+4N_f+2N'_f} + \dots)(v^2 - m^2)^{N_f''}t + v^{6N_f+4N'_f}(v^2 - m^2)^{2N_f''} = 0$$

where we ignored both the lower power terms in v and the scales for the gauge groups for simplicity.

From this curve of quartic equation for t above, the asymptotic regions can be classified as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{2N_c+2} + \dots \quad NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{2N_f+2N'_f+2N''_f-2\tilde{N}_c''-2} + \dots \quad NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{2N'_c-2N_c-2+2N_f} + \dots \quad NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{2\tilde{N}_c''+2-2N'_c+2N_f+2N'_f+2N''_f} + \dots \quad NS'_R \text{ asymptotic region.} \end{aligned}$$

3.3 Magnetic theory with dual for second gauge group

Let us take the Seiberg dual for the second gauge group factor $SO(2N'_c)$ while keeping the first and the third gauge group factors $Sp(N_c)$ and $Sp(N''_c)$ untouched. Suppose that $\Lambda_2 \gg \Lambda_1, \Lambda_3$. After we move the $NS5_R$ -brane in Figure 6 to the left all the way past the $NS5'_L$ -brane, the linking number of $NS5_R$ -brane from Figure 8A is given by $L_5 = -\frac{(2N'_f)}{2} - 1 - (1) + 2\tilde{N}'_c - 2N_c$. Originally, it was $L_5 = \frac{(2N'_f)}{2} + 1 - (-1) + 2N''_c - 2N'_c$ from Figure 6 in electric theory before the brane motion. Therefore, by the linking number conservation and equating these two L_5 's each other, we are left with the number of colors in the magnetic theory $\tilde{N}'_c = N'_f + N''_c + N_c - N'_c + 2$. Here the dependence on N_c and N''_c arises.

Let us draw this magnetic brane configuration in Figure 8A and we put the coincident N'_f D6-branes in the nonzero v direction (and its mirrors). For the N_f and N''_f D6-branes we consider the massless flavors and then they are located at $v = 0$. If we ignore the $NS5_L$ -brane, $2N_c$ D4-branes, $2N_f$ D6-branes, the $NS5'_R$ -brane, $2N''_c$ D4-branes and $2N''_f$ D6-branes (detaching these branes from Figure 8A), then this brane configuration leads to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SO(2\tilde{N}'_c = 2(N'_f + N''_c + N_c - N'_c + 2))$ with N'_f massive flavors [6, 11].

In the dual theory, since there exist $2N'_f$ fundamental fields q' , one bifundamental field g (which will give rise to the contribution of $2N''_c$), and one bifundamental field F (which will give rise to the contribution of $2N_c$), the coefficient of the beta function is $b_{SO(2\tilde{N}'_c)}^{mag} = 3(2\tilde{N}'_c - 2) - 2N'_f - 2N''_c - 2N_c$ and since there are $2\tilde{N}'_c$ fundamental fields g for the first factor, $2N''_f$ fundamental fields Q'' , an antisymmetric tensor Φ'' (which will contribute to $(2N''_c + 2)$), and a field X'' (which will contribute to $2N'_f$), the coefficient of the beta function is $b_{Sp(N''_c)}^{mag} = 3(2N''_c + 2) - 2\tilde{N}'_c - 2N''_f - (2N''_c + 2) - 2N'_f$. Since there exist $2N_f$ quarks Q , and one

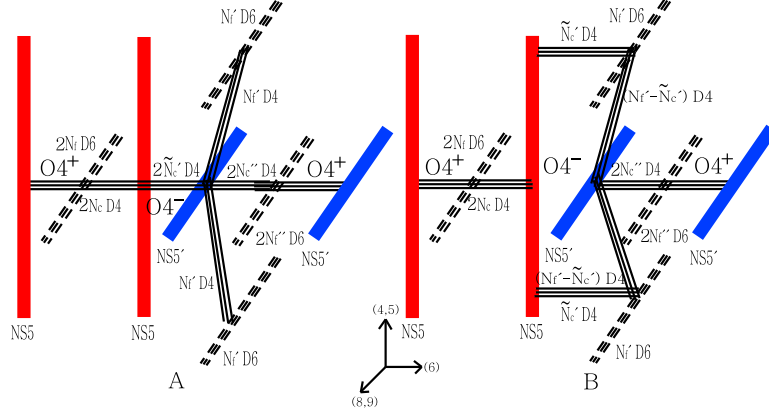


Figure 8: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $Sp(N_c) \times SO(2\tilde{N}'_c) \times Sp(N''_c)$ gauge group with flavors $Q(q')[Q'']$ for each gauge group, the bifundamentals $F(g)$, and gauge singlets in Figure 8A. In Figure 8B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q(Q'')$ is given.

bifundamental field F (which will give rise to the contribution of $2\tilde{N}'_c$), the coefficient of the beta function of the first gauge group factor is $b_{Sp(N_c)}^{mag} = 3(2N_c + 2) - 2N_f - 2\tilde{N}'_c$.

Therefore, both $Sp(N_c)$, $SO(2\tilde{N}'_c)$, and $Sp(N''_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b_{Sp(N_c)}^{mag} < 0$, $b_{SO(2\tilde{N}'_c)}^{mag} < 0$, and $b_{Sp(N''_c)}^{mag} < 0$. Neglecting the $Sp(N_c)$ and $Sp(N''_c)$ dynamics, the magnetic $SO(2\tilde{N}'_c)$ is IR free when

$$N'_c - N''_c - N_c - 2 < N'_f < \frac{3}{2}(N'_c - 1) - N''_c - N_c.$$

Then the dual gauge group and matter contents with the additional fields X'' , M' and Φ'' are

gauge group :	$Sp(N_c) \times SO(2\tilde{N}'_c) \times Sp(N''_c)$		
matter :	Q_f	$(\square, \mathbf{1}, \mathbf{1})$	$(f = 1, \dots, 2N_f)$
	$q'_{f'}$	$(\mathbf{1}, \square, \mathbf{1})$	$(f' = 1, \dots, 2N'_f)$
	$Q''_{f''}$	$(\mathbf{1}, \mathbf{1}, \square)$	$(f'' = 1, \dots, 2N''_f)$
	F	$(\square, \square, \mathbf{1})$	
	g	$(\mathbf{1}, \square, \square)$	
	$(X''_{n'} \equiv) GQ'$	$(\mathbf{1}, \mathbf{1}, \square)$	$(n' = 1, \dots, 2N'_f)$
	$(M'_{f',g'} \equiv) Q'Q'$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(f', g' = 1, \dots, 2N'_f)$
	$(\Phi'' \equiv) GG$	$(\mathbf{1}, \mathbf{1}, \text{symm})$	

The dual magnetic tree level superpotential, by adding the mass term for the Q' in electric theory corresponding to add a linear term in M' in dual magnetic theory, is given by

$$W_{dual} = (M'q'q' + gX''q' + \Phi''gg) + m'M'.$$

Here q' is fundamental for the gauge group index. Then, $q'q'$ has rank $2\tilde{N}'_c$ while m' has a rank $2N'_f$. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M' , cannot be satisfied if the rank $2N'_f$ exceeds $2\tilde{N}'_c$. This is so-called rank condition and the supersymmetry is broken [15].

The classical moduli space of vacua can be obtained from F-term equations. From the F-terms $F_{q'}$ and $F_{M'}$, one gets $M'q' + gX'' = 0 = q'q' + m'$. Similarly, one obtains $\Phi''g + X''q' = 0 = gg$ from the F-terms F_g and $F_{\Phi''}$. Moreover, there is a relation $q'g = 0$ from the F-term $F_{X''}$. Then, one obtains the following solutions

$$\langle q' \rangle = \begin{pmatrix} i\sqrt{m}\mathbf{1}_{2\tilde{N}'_c} \\ 0 \end{pmatrix}, \quad \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0\mathbf{1}_{2(N'_f - \tilde{N}'_c)} \end{pmatrix}, \quad \langle g \rangle = 0 = \langle X'' \rangle$$

where $M'_0\mathbf{1}_{2(N'_f - \tilde{N}'_c)}$ is an arbitrary $2(N'_f - \tilde{N}'_c) \times 2(N'_f - \tilde{N}'_c)$ symmetric matrix and the zeros of $\langle q' \rangle$ are $2(N'_f - \tilde{N}'_c) \times 2\tilde{N}'_c$ zero matrices. Similarly, the zeros of $2N'_f \times 2N'_f$ matrix M' are assumed also. As we did in previous case, one can analyze the one loop computation by expanding the fields around the vacua and it will lead to the fact that states are stable by realizing the mass of $m_{M'_0}^2$ positive.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 8B. If we ignore the $NS5_L$ -brane, $2N_c$ D4-branes, $2N_f$ D6-branes, the $NS5'_R$ -brane, $2N'_c$ D4-branes and $2N'_f$ D6-branes (detaching these branes from Figure 8B), then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SO(2\tilde{N}'_c)$ with N'_f massive flavors [6, 11].

The nonsupersymmetric minimal energy brane configuration Figure 8B with vanishing $2N'_f$ D6-branes leads to the Figure 3 of [15] with a reflection with respect to the NS5-brane if we ignore the $NS5_L$ -brane, $2N_f$ D6-branes and $2N_c$ D4-branes. Moreover, this Figure 8B with a replacement N'_f D6-branes by the NS5'-brane (neglecting the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes) will become the Figure 7B of [19] with a rotation of $NS5_R$ -brane by $\frac{\pi}{2}$ angle or the Figure 9B of [19] with a reflection with respect to the $NS5_R$ -brane if we ignore the $NS5'_R$ -brane, N'_f D6-branes and N'_c D4-branes from the Figure 8B.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5_L$ -brane to the right, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 8 with a reflection with respect to the NS5-brane.

After lifting the type IIA description to M-theory, the corresponding magnetic M5-brane configuration with equal mass for the quarks, in a background space of $xt = v^{2N_f+2N_f''} \prod_{k=1}^{N_f'} (v^2 - e_k^2)$ where this four dimensional space replaces (45610) directions, is characterized by [55]

$$t^4 + (v^{2N_c+2+2N_f} + \dots)t^3 + (v^{2\tilde{N}_c'+2N_f} + \dots)t^2 + (v^{2N_c''+2+4N_f} + \dots)t + v^{6N_f+2N_f''}(v^2 - m^2)^{N_f'} = 0$$

where we ignored both the lower power terms in v and the scales for the gauge groups for simplicity.

From this curve of quartic equation for t above, the asymptotic regions can be classified by as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{2N_c+2+2N_f} + \dots & NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{2\tilde{N}_c'-2N_c-2} + \dots & NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{2N_c''+2-2\tilde{N}_c'+2N_f} + \dots & NS_L' \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{2N_f+2N_f'+2N_f''-2N_c''-2} + \dots & NS_R' \text{ asymptotic region.} \end{aligned}$$

3.4 Magnetic theory with dual for second gauge group

Let us take the Seiberg dual for the second gauge group factor $SO(2N_c')$ while keeping the first and the third gauge group factors $Sp(N_c)$ and $Sp(N_c'')$ untouched. Suppose that $\Lambda_2 \gg \Lambda_1, \Lambda_3$.

Let us move the $NS5_L'$ -brane in Figure 6 to the right all the way past the $NS5_R$ -brane. After this brane motion, one arrives at the Figure 9A. We need to change the Figure 6 a little bit such that the N_f' D6-branes are not parallel to the $NS5_L'$ -brane. That is, we rotate N_f' D6-branes a little bit (this does not change the classical electric superpotential as we explained before) and after dualizing the second gauge group, we rotate those N_f' D6-branes with opposite direction. The linking number of $NS5_L'$ -brane from Figure 9A is given by $L_5 = \frac{(2N_f')}{2} + 1 - (-1) - 2\tilde{N}_c' + 2N_c''$. Originally, it was $L_5 = -\frac{(2N_f')}{2} + (-1) - (+1) + 2N_c' - 2N_c$ from Figure 6 before the brane motion. Therefore, by the linking number conservation and equating these two L_5 's each other, we are left with the number of colors in the magnetic theory $\tilde{N}_c' = N_f' + N_c'' + N_c - N_c' + 2$.

Let us draw this magnetic brane configuration in Figure 9A and we put the coincident N_f' D6-branes in the nonzero v direction (and its mirrors).

contribute to $2N_f$), the coefficient of the beta function of the first gauge group factor is $b_{Sp(N_c)}^{mag} = 3(2N_c + 2) - 2N_f - 2\tilde{N}'_c - (2N_c + 2) - 2N_f$.

Therefore, both $Sp(N_c)$, $SO(2\tilde{N}'_c)$, and $Sp(N''_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b_{Sp(N_c)}^{mag} < 0$, $b_{SO(2\tilde{N}'_c)}^{mag} < 0$, and $b_{Sp(N''_c)}^{mag} < 0$. Neglecting the $Sp(N_c)$ and $Sp(N''_c)$ dynamics, the magnetic $SO(2\tilde{N}'_c)$ is IR free when

$$N'_c - N''_c - N_c - 2 < N'_f < \frac{3}{2}(N'_c - 1) - N''_c - N_c.$$

The dual magnetic tree level superpotential, by adding the mass term for the Q' in electric theory corresponding to add a linear term in M' in dual magnetic theory, is given by

$$W_{dual} = (M'q'q' + fXq' + \Phi ff) + m'M'.$$

Here q' is fundamental for the gauge group index. Then, $q'q'$ has rank $2\tilde{N}'_c$ while m' has a rank $2N'_f$. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M' , cannot be satisfied if the rank $2N'_f$ exceeds $2\tilde{N}'_c$. This is so-called rank condition [15] and the supersymmetry is broken.

The classical moduli space of vacua can be obtained from F-term equations. From the F-terms $F_{q'}$ and $F_{M'}$, one gets $M'q' + fX = 0 = q'q' + m'$. Similarly, one obtains $\Phi f + Xq' = 0 = ff$ from the F-terms F_f and F_Φ . Moreover, there is a relation $q'f = 0$ from the F-term F_X . Then, one obtains the following solutions

$$\langle q' \rangle = \begin{pmatrix} i\sqrt{m}\mathbf{1}_{2\tilde{N}'_c} \\ 0 \end{pmatrix}, \quad \langle M' \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M'_0\mathbf{1}_{2(N'_f - \tilde{N}'_c)} \end{pmatrix}, \quad \langle f \rangle = 0 = \langle X \rangle$$

where $M'_0\mathbf{1}_{2(N'_f - \tilde{N}'_c)}$ is an arbitrary $2(N'_f - \tilde{N}'_c) \times 2(N'_f - \tilde{N}'_c)$ symmetric matrix and the zeros of $\langle q' \rangle$ are $2(N'_f - \tilde{N}'_c) \times 2\tilde{N}'_c$ zero matrices. Similarly the zeros of $2N'_f \times 2N'_f$ matrix M' are assumed also. As we did in previous case, one can analyze the one loop computation by expanding the fields around the vacua and it will lead to the fact that states are stable by realizing the mass of $m_{M'_0}^2$ positive.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 9B. If we ignore the $NS5_L$ -brane, $2N_c$ D4-branes, $2N_f$ D6-branes, $NS5'_R$ -brane, $2N''_c$ D4-branes and $2N''_f$ D6-branes(detaching these from Figure 9B), as observed already, then this brane configuration looks similar to the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SO(2\tilde{N}'_c = 2(N'_f - N'_c + 2))$ with N'_f massive flavors. The difference occurs in the position of D6-branes.

The nonsupersymmetric minimal energy brane configuration Figure 9B with a replacement N'_f D6-branes by the NS5'-brane (neglecting the $NS5'_R$ -brane, $2N''_f$ D6-branes and $2N'_c$ D4-branes with vanishing $2N_f$ D6-branes) leads to the Figure 10B of [19] with a rotation of $NS5'_L$ -brane by $\frac{\pi}{2}$ angle.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5'_R$ -brane to the left, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 9 with a reflection with respect to the NS5-brane.

After lifting the type IIA description we explained so far to M-theory, the corresponding magnetic M5-brane configuration, in a background space of $xt = v^{2N_f+2N''_f} \prod_{k=1}^{N'_f} (v^2 - e_k^2)$ where this four dimensional space replaces (45610) directions, is characterized by [55]

$$t^4 + (v^{2N_c+2} + \dots)t^3 + \left[v^{2\tilde{N}'_c+2N_f}(v^2 - m^2)^{N'_f} + \dots \right] t^2 + \left[v^{2N''_c+2+4N_f}(v^2 - m^2)^{2N'_f} + \dots \right] t + v^{6N_f+2N''_f}(v^2 - m^2)^{3N'_f} = 0.$$

From this curve of quartic equation for t above, the asymptotic regions can be classified as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{2N_c+2} + \dots \quad NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{2\tilde{N}'_c-2N_c-2+2N_f+2N'_f} + \dots \quad NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{2N''_c+2-2\tilde{N}'_c+2N_f+2N'_f} + \dots \quad NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{2N_f+2N'_f+2N''_f-2N'_c-2} + \dots \quad NS'_R \text{ asymptotic region.} \end{aligned}$$

3.5 Magnetic theory with dual for first gauge group

Let us take the Seiberg dual for the first gauge group factor $Sp(N_c)$ while keeping the second and the third gauge group factors $SO(2N'_c)$ and $Sp(N'_c)$ untouched. Suppose that $\Lambda_1 \gg \Lambda_2, \Lambda_3$. After we move the $NS5_L$ -brane in Figure 6 to the right all the way past the $NS5'_L$ -brane, one arrives at Figure 10A and the linking number of $NS5_L$ -brane from Figure 10A is given by $L_5 = \frac{(2N_f)}{2} - 1 - (1) - 2\tilde{N}'_c + 2N'_c$. Originally, it was $L_5 = -\frac{(2N_f)}{2} + 1 - (-1) + 2N_c$ from Figure 6 before the brane motion. Therefore, by the linking number conservation and equating these two L_5 's each other, we are left with the number of colors in the magnetic theory $\tilde{N}_c = N_f + N'_c - N_c - 2$.

Let us draw this magnetic brane configuration in Figure 10A and we put the coincident N_f D6-branes in the nonzero v direction (and its mirrors). If we ignore the $NS5_R$ -brane, N'_c

D4-branes, N'_f D6-branes, the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes (detaching these branes from Figure 10A), then this brane configuration leads to the standard $\mathcal{N} = 1$ SQCD with the magnetic gauge group $Sp(\tilde{N}_c = N_f - N_c - 2)$ with N_f massive flavors [6, 11].

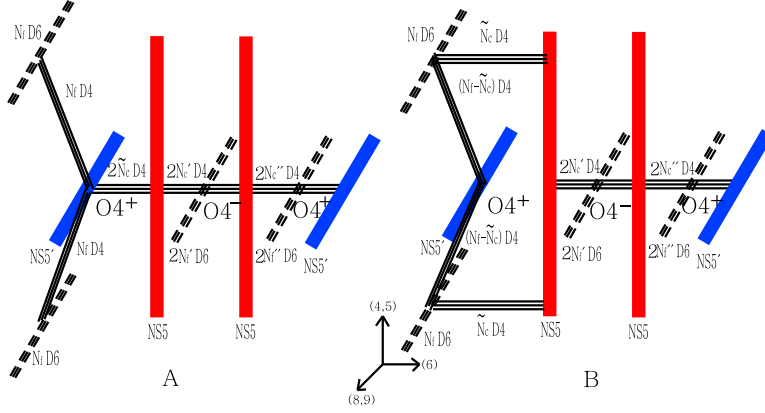


Figure 10: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration with $Sp(\tilde{N}_c) \times SO(2N'_c) \times Sp(N''_c)$ gauge group with fundamentals $q(Q')[Q'']$ for each gauge group, the bi-fundamentals $F(g)$, and gauge singlets in Figure 10A. In Figure 10B, the nonsupersymmetric minimal energy brane configuration with the same gauge group and matter contents above for massless $Q'(Q'')$ is given.

In the dual theory, since there exist $2N_f$ fundamental fields q , and one bifundamental field f (which will give rise to the contribution of $2N'_c$), the coefficient of the beta function is $b_{Sp(\tilde{N}_c)}^{mag} = 3(2\tilde{N}_c + 2) - 2N_f - 2N'_c$ and since there are $2N'_f$ fundamental fields Q' , $2\tilde{N}_c$ fundamental fields f for the second factor, $2N''_c$ fundamental fields G for the flavor index of the first factor, an antisymmetric tensor Φ' (which will contribute to $(2N'_c - 2)$), and $2N_f$ fields X' , the coefficient of the beta function is $b_{SO(2N'_c)}^{mag} = 3(2N'_c - 2) - 2N'_f - 2\tilde{N}_c - 2N''_c - (2N'_c - 2) - 2N_f$. Since there exist $2N''_f$ quarks Q'' , and one bifundamental field G (which will give rise to the contribution of $2N'_c$), the coefficient of the beta function of the third gauge group factor is $b_{Sp(N''_c)}^{mag} = 3(2N''_c + 2) - 2N''_f - 2N'_c = b_{Sp(N''_c)}$.

Therefore, both $Sp(\tilde{N}_c)$, $SO(2N'_c)$, and $Sp(N''_c)$ gauge couplings are IR free by requiring the negativeness of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b_{Sp(\tilde{N}_c)}^{mag} < 0$, $b_{SO(2N'_c)}^{mag} < 0$, and $b_{Sp(N''_c)}^{mag} < 0$. Note that the $SO(2N'_c)$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SO(2N'_c)} - b_{SO(2N'_c)}^{mag} > 0$, $SO(2N'_c)$ is more asymptotically free than $SO(2N'_c)^{mag}$. Neglecting the $SO(2N'_c)$ and $Sp(N''_c)$ dynamics, the magnetic $Sp(\tilde{N}_c)$ is IR free when

$$N_c - N'_c + 2 < N_f < \frac{3}{2}(N_c + 1) - N'_c.$$

Then the dual gauge group and matter contents with the additional fields X', M and Φ' are

gauge group :	$Sp(\tilde{N}_c) \times SO(2N'_c) \times Sp(N''_c)$		
matter :	q_f	$(\square, \mathbf{1}, \mathbf{1})$	$(f = 1, \dots, 2N_f)$
	$Q'_{f'}$	$(\mathbf{1}, \square, \mathbf{1})$	$(f' = 1, \dots, 2N'_f)$
	$Q''_{f''}$	$(\mathbf{1}, \mathbf{1}, \square)$	$(f'' = 1, \dots, 2N''_f)$
	f	$(\square, \square, \mathbf{1})$	
	G	$(\mathbf{1}, \square, \square)$	
	$(X'_n \equiv) FQ$	$(\mathbf{1}, \square, \mathbf{1})$	$(n = 1, \dots, 2N_f)$
	$(M_{f,g} \equiv) QQ$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(f, g = 1, \dots, 2N_f)$
	$(\Phi' \equiv) FF$	$(\mathbf{1}, \text{asymm}, \mathbf{1})$	

The dual magnetic tree level superpotential, by adding the mass term for the Q in electric theory corresponding to add a linear term in M in dual magnetic theory, is given by [15]

$$W_{dual} = (Mqq + fX'q + \Phi'ff) + mM.$$

Here q is fundamental for the gauge group index. Then, qq has rank $2\tilde{N}_c$ while m has a rank $2N_f$. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M , cannot be satisfied if the rank $2N_f$ exceeds $2\tilde{N}_c$. This is so-called rank condition and the supersymmetry is broken.

The classical moduli space of vacua can be obtained from F-term equations. From the F-terms F_q and F_M , one gets $Mq + fX' = 0 = qq + m$. Similarly, one obtains $\Phi'f + X'q = 0 = ff$ from the F-terms F_f and $F_{\Phi'}$. Moreover, there is a relation $qf = 0$ from the F-term $F_{X'}$. Then, one obtains the following solutions

$$\langle q \rangle = \begin{pmatrix} i\sqrt{m}\mathbf{1}_{2\tilde{N}_c} \\ 0 \end{pmatrix}, \langle M \rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_0\mathbf{1}_{(N_f - \tilde{N}_c)} \otimes i\sigma^2 \end{pmatrix}, \langle f \rangle = 0 = \langle X' \rangle$$

where $M_0\mathbf{1}_{(N_f - \tilde{N}_c)} \otimes i\sigma^2$ is an arbitrary $2(N_f - \tilde{N}_c) \times 2(N_f - \tilde{N}_c)$ antisymmetric matrix and the zeros of $\langle q \rangle$ are $2(N_f - \tilde{N}_c) \times 2\tilde{N}_c$ zero matrices. Similarly, the zeros of $2N_f \times 2N_f$ matrix M are assumed also. As we did in previous case, one can analyze the one loop computation by expanding the fields around the vacua and it will lead to the fact that states are stable by realizing the mass of $m_{M_0}^2$ positive.

Then the minimal energy supersymmetry breaking brane configuration is shown in Figure 10B. If we ignore the $NS5_R$ -brane, $2N'_c$ D4-branes, $2N'_f$ D6-branes, $NS5'_R$ -brane, $2N''_c$ D4-branes and $2N''_f$ D6-branes(detaching these from Figure 10B), as observed already, then this

brane configuration is the minimal energy supersymmetry breaking brane configuration for the $\mathcal{N} = 1$ SQCD with the magnetic gauge group $Sp(\tilde{N}_c = N_f - N_c - 2)$ with N_f massive flavors [6, 11].

The nonsupersymmetric minimal energy brane configuration Figure 10B with a replacement N_f D6-branes by the NS5'-brane(neglecting the $NS5'_R$ -brane, $2N_f''$ D6-branes and $2N_c''$ D4-branes with vanishing $2N_f'$ D6-branes) leads to the Figure 10B of [19].

When we move the $NS5'_L$ -brane in Figure 6 to the left all the way past the $NS5_L$ -brane, then one arrives at the magnetic brane configuration similar to the Figure 10. The only difference is that the N_f D6-branes are located at the right hand side of the $NS5_L$ -brane. Then this nonsupersymmetric minimal energy brane configuration with a replacement N_f D6-branes by the NS5'-brane(neglecting the $NS5'_R$ -brane, $2N_f''$ D6-branes and $2N_c''$ D4-branes) leads to the Figure 10B of [19] with a reflection with respect to the NS5-brane and an extra rotation of $NS5'_L$ -brane by $\frac{\pi}{2}$ angle.

Starting with $NS5'_L$ - $NS5_L$ - $NS5'_R$ - $NS5_R$ branes configuration, as in the footnote 3, by moving the $NS5_L$ -brane to the left, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the Figure 7 with a reflection with respect to the NS5-brane. Furthermore, by moving the $NS5'_R$ -brane to the right, one gets the nonsupersymmetric minimal energy brane configuration which is exactly the new Figure in previous paragraph with a reflection with respect to the $NS5_L$ -brane.

After lifting the type IIA description we explained so far to M-theory, the corresponding magnetic M5-brane configuration, in a background space of $xt = v^{2N_f'+2N_f''} \prod_{k=1}^{N_f} (v^2 - e_k^2)$ where this four dimensional space replaces (45610) directions, is characterized by [55]

$$t^4 + (v^{2\tilde{N}_c+2} + \dots)t^3 + \left[v^{2N'_c}(v^2 - m^2)^{N_f} + \dots \right] t^2 + \left[v^{2N_c''+2+2N'_f}(v^2 - m^2)^{2N_f} + \dots \right] t + (v^2 - m^2)^{3N_f} v^{4N'_f+2N_f''} = 0.$$

From this curve of quartic equation for t above, the asymptotic regions can be classified as follows:

1. $v \rightarrow \infty$ limit implies

$$\begin{aligned} w \rightarrow 0, \quad y &\sim v^{2N'_c-2\tilde{N}_c-2+2N_f} + \dots \quad NS_L \text{ asymptotic region,} \\ w \rightarrow 0, \quad y &\sim v^{2N_f+2N'_f+2N'_c+2-2N'_c} + \dots \quad NS_R \text{ asymptotic region.} \end{aligned}$$

2. $w \rightarrow \infty$ limit implies

$$\begin{aligned} v \rightarrow m, \quad y &\sim w^{2\tilde{N}_c+2} + \dots \quad NS'_L \text{ asymptotic region,} \\ v \rightarrow m, \quad y &\sim w^{2N_f+2N'_f+2N'_c-2N_c''-2} + \dots \quad NS'_R \text{ asymptotic region.} \end{aligned}$$

3.6 Other magnetic theories with opposite O4-plane charges

By changing the charges of O4-plane in previous brane configuration given in Figure 6, the type IIA brane configuration is realized by an $\mathcal{N} = 1$ supersymmetric gauge theory with [54]

$$SO(2N_c) \times Sp(N'_c) \times SO(2N''_c)$$

and corresponding matter contents. Then by deforming this electric theory by mass term for the quarks and taking the magnetic dual on each gauge group factor, one gets the possible meta-stable brane configurations.

There exists an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with

$$SO(2N_c) \times Sp(N'_c) \times SO(2\tilde{N}''_c), \quad \tilde{N}''_c \equiv N''_f + N'_c - N''_c + 2$$

with matters and the corresponding brane configuration is given by the Figure 7 with opposite O4-plane charges($O4^+$ goes to $O4^-$ and vice versa). The constant term $+2$ in the dual color has opposite sign, compared with the discussion of subsection 3.2 where it has -2 . This is due to the fact that the linking number counting in this case has opposite sign for the contribution of O4-plane since we have different charge in the electric and magnetic theories. Since the dual gauge group is an orthogonal gauge group, the corresponding gauge singlet M'' is a symmetric two index tensor in the flavor indices(leading to different structure of vacuum expectation value, compared with the solution in the subsection 3.2) and the field Φ' is a symmetric under the second gauge group.

Also there is an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with

$$SO(2N_c) \times Sp(\tilde{N}'_c) \times SO(2N''_c), \quad \tilde{N}'_c \equiv N'_f + N''_c + N_c - N'_c - 2$$

with matters and the corresponding brane configuration is given by the Figure 8 with opposite O4-plane charges($O4^+$ goes to $O4^-$ and vice versa). Since the dual gauge group is a symplectic gauge group, the corresponding gauge singlet M' is an antisymmetric two index tensor in the flavor indices(leading to different structure of vacuum expectation value, compared with the solution in the subsection 3.3) and the field Φ'' is an antisymmetric under the third gauge group.

Moreover, there is an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with

$$SO(2N_c) \times Sp(\tilde{N}'_c) \times SO(2N''_c), \quad \tilde{N}'_c \equiv N'_f + N''_c + N_c - N'_c - 2$$

with matters and the corresponding brane configuration is given by the Figure 9 with opposite O4-plane charges($O4^+$ goes to $O4^-$ and vice versa). Since the dual gauge group is a symplectic

gauge group, the corresponding gauge singlet M' is an antisymmetric two index tensor in the flavor indices(leading to different structure of vacuum expectation value, compared with the solution in the subsection 3.4) and the field Φ is an antisymmetric under the first gauge group.

Finally, there exists an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with

$$SO(2\tilde{N}_c) \times Sp(N'_c) \times SO(2N''_c), \quad \tilde{N}_c \equiv N_f + N'_c - N_c + 2$$

with matters and the corresponding brane configuration is given by the Figure 10 with opposite $O4$ -plane charges($O4^+$ goes to $O4^-$ and vice versa). Since the dual gauge group is a symplectic gauge group, the corresponding gauge singlet M is a symmetric two index tensor in the flavor indices(leading to different structure of vacuum expectation value, compared with the solution in the subsection 3.5) and the field Φ' is a symmetric under the second gauge group.

The remaining analysis can be done easily without any difficulty.

3.7 Magnetic theories for the multiple product gauge groups

3.7.1 The symplectic gauge groups both at the start and end of the chain

Now one can generalize the method for the triple product gauge groups to the finite multiple product gauge groups characterized by [54]

$$Sp(N_{c,1}) \times SO(2N_{c,2}) \times \cdots \times SO(2N_{c,n-1}) \times Sp(N_{c,n})$$

with the $(n-1)$ bifundamentals $(\square_1, \square_2, \mathbf{1}, \cdots, \mathbf{1}), \cdots$, and $(\mathbf{1}, \cdots, \mathbf{1}, \square_{n-1}, \square_n)$, and n -quarks $(\square_1, \mathbf{1}, \cdots, \mathbf{1}), \cdots$, and $(\mathbf{1}, \cdots, \mathbf{1}, \square_n)$. Note that there are the symplectic gauge groups both at the start and end of the chain. Then the electric superpotential is $W_{elec} = \sum_{i=1}^n Q_i Q_i$.

There exist $(2n-2)$ magnetic theories and they can be classified as six cases as follows.

- Case 1

When the Seiberg dual is taken for the first gauge group factor by assuming that $\Lambda_1 \gg \Lambda_i$ where $i = 2, \cdots, n$, one follows the procedure given in the subsection 3.5. The gauge group is

$$Sp(\tilde{N}_{c,1} \equiv N_{f,1} + N_{c,2} - N_{c,1} - 2) \times SO(2N_{c,2}) \times \cdots \times Sp(N_{c,n})$$

and the matter contents are given by the dual quark q_1 in the representation $(\square_1, \mathbf{1}, \cdots, \mathbf{1})$ as well as $(n-1)$ quarks Q_i where $i = 2, \cdots, n$, the bifundamentals f_1 in the representation $(\square_1, \square_2, \mathbf{1}, \cdots, \mathbf{1})$ under the gauge group, \cdots in addition to $(n-2)$ bifundamentals G_i , and various gauge singlets X_2, M_1 and Φ_2 . The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 10. The

magnetic superpotential is $W_{dual} = (M_1 q_1 q_1 + f_1 X_2 q_1 + \Phi_2 f_1 f_1) + m_1 M_1$. By computing the contribution for the one loop as in the subsection 3.5, the vacua are stable and the asymptotic behavior of $(n + 1)$ NS-branes can be obtained also.

- Case 2

When the Seiberg dual is taken for the last gauge group factor by assuming that $\Lambda_n \gg \Lambda_i$ where $i = 1, 2, \dots, (n - 1)$, one follows the procedure given in the subsection 3.2. The gauge group is given by

$$Sp(N_{c,1}) \times \dots \times SO(2N_{c,n-1}) \times Sp(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n} - 2).$$

The corresponding brane configuration can be obtained similarly and the extra $(n - 3)$ NS-branes, $(n - 3)$ sets of D6-branes and $(n - 3)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane of Figure 7. The magnetic superpotential is $W_{dual} = (M_n q_n q_n + g_{n-1} X_{n-1} q_n + \Phi_{n-1} g_{n-1} g_{n-1}) + m_n M_n$.

- Case 3

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i - 1, i + 1, \dots, n$, one follows the procedure given in the subsection 3.3. The gauge group is given by

$$\dots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \dots$$

where $i = 2, 4, \dots, (n - 1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i - 2)$ NS-branes, $(i - 2)$ sets of D6-branes and $(i - 2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n - i - 1)$ NS-branes, $(n - i - 1)$ sets of D6-branes and $(n - i - 1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$.

- Case 4

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i - 1, i + 1, \dots, n$, one follows the procedure given in the subsection 3.4. The gauge group is given by

$$\dots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \dots$$

where $i = 2, 4, \dots, (n - 1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i - 2)$ NS-branes, $(i - 2)$ sets of D6-branes and $(i - 2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n - i - 2)$ NS-branes, $(n - i - 2)$ sets of D6-branes and

$(n - i - 2)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$.

• Case 5

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i - 1, i + 1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n - 2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i - 2)$ NS-branes, $(i - 2)$ sets of D6-branes and $(i - 2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n - i - 1)$ NS-branes, $(n - i - 1)$ sets of D6-branes and $(n - i - 1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$.

• Case 6

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i - 1, i + 1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n - 2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i - 2)$ NS-branes, $(i - 2)$ sets of D6-branes and $(i - 2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n - i - 1)$ NS-branes, $(n - i - 1)$ sets of D6-branes and $(n - i - 1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$.

3.7.2 The orthogonal gauge groups both at the start and end of the chain

When the electric theory is described by

$$SO(2N_{c,1}) \times Sp(N_{c,2}) \times \cdots \times Sp(N_{c,n-1}) \times SO(2N_{c,n})$$

with the $(n - 1)$ bifundamentals $(\square_1, \square_2, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \square_{n-1}, \square_n)$, and n -quarks in the representation $(\square_1, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \square_n)$, there exist $(2n - 2)$ magnetic theories and they can be classified as six cases as follows. Note that there are the orthogonal gauge groups both at the start and end of the chain.

• Case 1'

When the Seiberg dual is taken for the first gauge group factor by assuming that $\Lambda_1 \gg \Lambda_i$ where $i = 2, \dots, n$, one follows the procedure given in the subsection 3.5. The gauge group is

$$SO(2\tilde{N}_{c,1} \equiv 2(N_{f,1} + N_{c,2} - N_{c,1} + 2)) \times Sp(N_{c,2}) \times \dots \times SO(2N_{c,n})$$

and the matter contents are given by the dual quark q_1 in the representation $(\square_1, \mathbf{1}, \dots, \mathbf{1})$ as well as $(n-1)$ quarks Q_i where $i = 2, \dots, n$, the bifundamentals f_1 in the representation $(\square_1, \square_2, \mathbf{1}, \dots, \mathbf{1})$, \dots in addition to $(n-2)$ bifundamentals G_i , and various gauge singlets X_2, M_1 and Φ_2 . The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 10. The magnetic superpotential is $W_{dual} = (M_1 q_1 q_1 + f_1 X_2 q_1 + \Phi_2 f_1 f_1) + m_1 M_1$.

• Case 2'

When the Seiberg dual is taken for the last gauge group factor by assuming that $\Lambda_n \gg \Lambda_i$ where $i = 1, 2, \dots, (n-1)$, one follows the procedure given in the subsection 3.2. The gauge group is given by

$$SO(2N_{c,1}) \times \dots \times Sp(N_{c,n-1}) \times SO(2\tilde{N}_{c,n} \equiv 2(N_{f,n} + N_{c,n-1} - N_{c,n} + 2)).$$

The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane of Figure 7. The magnetic superpotential is $W_{dual} = (M_n q_n q_n + g_{n-1} X_{n-1} q_n + \Phi_{n-1} g_{n-1} g_{n-1}) + m_n M_n$.

• Case 3'

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.3. The gauge group is given by

$$\dots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \dots$$

where $i = 2, 4, \dots, (n-1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$.

• Case 4'

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.4. The gauge group is given by

$$\cdots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \cdots$$

where $i = 2, 4, \dots, (n-1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$.

• Case 5'

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n-2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$.

• Case 6'

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n-2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$.

3.7.3 The symplectic gauge group at the start and the orthogonal gauge group at end of the chain

When the electric theory is described by

$$Sp(N_{c,1}) \times SO(2N_{c,2}) \times \cdots \times Sp(N_{c,n-1}) \times SO(2N_{c,n})$$

with the $(n-1)$ bifundamentals $(\square_1, \square_2, \mathbf{1}, \cdots, \mathbf{1}), \cdots$, and $(\mathbf{1}, \cdots, \mathbf{1}, \square_{n-1}, \square_n)$, and n -quarks in the representation $(\square_1, \mathbf{1}, \cdots, \mathbf{1}), \cdots$, and $(\mathbf{1}, \cdots, \mathbf{1}, \square_n)$, there exist $(2n-2)$ magnetic theories and they can be classified as six cases as follows. Note that there are the symplectic gauge group at the start and the orthogonal gauge group at the end of the chain.

- Case 1''

When the Seiberg dual is taken for the first gauge group factor by assuming that $\Lambda_1 \gg \Lambda_i$ where $i = 2, \cdots, n$, one follows the procedure given in the subsection 3.5. The gauge group is

$$Sp(\tilde{N}_{c,1} \equiv N_{f,1} + N_{c,2} - N_{c,1} - 2) \times SO(2N_{c,2}) \times \cdots \times SO(2N_{c,n})$$

and the matter contents are given by the dual quark q_1 in the representation $(\square_1, \mathbf{1}, \cdots, \mathbf{1})$ as well as $(n-1)$ quarks Q_i where $i = 2, \cdots, n$, the bifundamentals f_1 in the representation $(\square_1, \square_2, \mathbf{1}, \cdots, \mathbf{1}), \cdots$ in addition to $(n-2)$ bifundamentals G_i , and various gauge singlets X_2, M_1 and Φ_2 . The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 10. The magnetic superpotential is $W_{dual} = (M_1 q_1 q_1 + f_1 X_2 q_1 + \Phi_2 f_1 f_1) + m_1 M_1$.

- Case 2''

When the Seiberg dual is taken for the last gauge group factor by assuming that $\Lambda_n \gg \Lambda_i$ where $i = 1, 2, \cdots, (n-1)$, one follows the procedure given in the subsection 3.2. The gauge group is given by

$$Sp(N_{c,1}) \times \cdots \times Sp(N_{c,n-1}) \times SO(2\tilde{N}_{c,n} \equiv 2(N_{f,n} + N_{c,n-1} - N_{c,n} + 2)).$$

The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane of Figure 7. The magnetic superpotential is $W_{dual} = (M_n q_n q_n + g_{n-1} X_{n-1} q_n + \Phi_{n-1} g_{n-1} g_{n-1}) + m_n M_n$.

- Case 3''

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \cdots, i-1, i+1, \cdots, n$, one follows the procedure given in the

subsection 3.3. The gauge group is given by

$$\cdots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \cdots$$

where $i = 2, 4, \dots, (n-1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$.

- Case 4''

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.4. The gauge group is given by

$$\cdots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \cdots$$

where $i = 2, 4, \dots, (n-1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$.

- Case 5''

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n-2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$.

- Case 6''

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n-2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$.

3.7.4 The orthogonal gauge group at the start and the symplectic gauge group at end of the chain

When the electric theory is described by

$$SO(2N_{c,1}) \times Sp(N_{c,2}) \times \cdots \times SO(2N_{c,n-1}) \times Sp(N_{c,n})$$

with the $(n-1)$ bifundamentals $(\square_1, \square_2, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \square_{n-1}, \square_n)$, and n -quarks in the representation $(\square_1, \mathbf{1}, \dots, \mathbf{1}), \dots$, and $(\mathbf{1}, \dots, \mathbf{1}, \square_n)$, there exist $(2n-2)$ magnetic theories and they can be classified as six cases as follows. Note that there are the orthogonal gauge group at the start and the symplectic gauge group at the end of the chain.

- Case 1'''

When the Seiberg dual is taken for the first gauge group factor by assuming that $\Lambda_1 \gg \Lambda_i$ where $i = 2, \dots, n$, one follows the procedure given in the subsection 3.5. The gauge group is

$$SO(2\tilde{N}_{c,1} \equiv 2(N_{f,1} + N_{c,2} - N_{c,1} + 2)) \times Sp(N_{c,2}) \times \cdots \times Sp(N_{c,n})$$

and the matter contents are given by the dual quark q_1 in the representation $(\square_1, \mathbf{1}, \dots, \mathbf{1})$ as well as $(n-1)$ quarks Q_i where $i = 2, \dots, n$, the bifundamentals f_1 in the representation $(\square_1, \square_2, \mathbf{1}, \dots, \mathbf{1}), \dots$ in addition to $(n-2)$ bifundamentals G_i , and various gauge singlets X_2, M_1 and Φ_2 . The magnetic superpotential is $W_{dual} = (M_1 q_1 q_1 + f_1 X_2 q_1 + \Phi_2 f_1 f_1) + m_1 M_1$. The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 10.

- Case 2'''

When the Seiberg dual is taken for the last gauge group factor by assuming that $\Lambda_n \gg \Lambda_i$ where $i = 1, 2, \dots, (n-1)$, one follows the procedure given in the subsection 3.2. The gauge group is given by

$$SO(2N_{c,1}) \times \dots \times SO(2N_{c,n-1}) \times Sp(\tilde{N}_{c,n} \equiv N_{f,n} + N_{c,n-1} - N_{c,n} - 2).$$

The magnetic superpotential is $W_{dual} = (M_n q_n q_n + g_{n-1} X_{n-1} q_n + \Phi_{n-1} g_{n-1} g_{n-1}) + m_n M_n$. The corresponding brane configuration can be obtained similarly and the extra $(n-3)$ NS-branes, $(n-3)$ sets of D6-branes and $(n-3)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane of Figure 7.

- Case 3'''

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.3. The gauge group is given by

$$\dots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \dots$$

where $i = 2, 4, \dots, (n-1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8.

- Case 4'''

When the Seiberg dual is taken for the middle gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.4. The gauge group is given by

$$\dots \times Sp(N_{c,i-1}) \times SO(2\tilde{N}_{c,i} \equiv 2(N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} + 2)) \times Sp(N_{c,i+1}) \times \dots$$

where $i = 2, 4, \dots, (n-1)$ implying that the number of possible magnetic gauge group is $\frac{n-1}{2}$. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9.

- Case 5'''

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n-2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + g_i X_{i+1} q_i + \Phi_{i+1} g_i g_i) + m_i M_i$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 8.

• Case 6'''

When the Seiberg dual is taken for the symplectic gauge group factor by assuming that $\Lambda_i \gg \Lambda_j$ where $j = 1, 2, \dots, i-1, i+1, \dots, n$, one follows the procedure given in the subsection 3.6. The gauge group is given by

$$\cdots \times SO(2N_{c,i-1}) \times Sp(\tilde{N}_{c,i} \equiv N_{f,i} + N_{c,i+1} + N_{c,i-1} - N_{c,i} - 2) \times SO(2N_{c,i+1}) \times \cdots$$

where $i = 3, 5, \dots, (n-2)$ implying that the number of possible magnetic gauge group is $\frac{n-3}{2}$. The magnetic superpotential is $W_{dual} = (M_i q_i q_i + f_{i-1} X_{i-1} q_i + \Phi_{i-1} f_{i-1} f_{i-1}) + m_i M_i$. The corresponding brane configuration can be obtained similarly and the extra $(i-2)$ NS-branes, $(i-2)$ sets of D6-branes and $(i-2)$ sets of D4-branes are present at the left hand side of the $NS5_L$ -brane and the extra $(n-i-1)$ NS-branes, $(n-i-1)$ sets of D6-branes and $(n-i-1)$ sets of D4-branes are present at the right hand side of the $NS5'_R$ -brane of Figure 9.

4 Conclusions and outlook

The meta-stable brane configurations we have found are summarized by Figures 2B, 3B, 4B, 5B (and 7B, 8B, 9B, and 10B). The nonsupersymmetric minimal energy brane configurations in Figure 2B and 3B lead to the Figure 3 of [14] if we ignore the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes. Similarly the nonsupersymmetric minimal energy brane configurations in Figure 7B and 8B lead to the Figure 3 of [15] if we ignore the $NS5_L$ -brane, $2N_f$ D6-branes and $2N_c$ D4-branes.

The Figures 2B and 7B with a replacement N_f'' D6-branes by the $NS5'$ -brane by neglecting the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes become the Figures 2B and 7B of [19] together with a reflection with respect to the $NS5_L$ -brane and a rotation of $NS5_R$ -brane by

$\frac{\pi}{2}$ angle respectively. The Figures 3B and 8B with a replacement N'_f D6-branes by the NS5'-brane by neglecting the $NS5_L$ -brane, N_f D6-branes and N_c D4-branes become the Figures 2B and 7B of [19] with a rotation of $NS5_R$ -brane by $\frac{\pi}{2}$ angle respectively or the Figures 4B and 9B of [19] with a reflection with respect to the $NS5_R$ -brane if we ignore the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes from the Figure 3B(8B) respectively. The Figures 4B and 9B with a replacement N'_f D6-branes by the NS5'-brane by neglecting the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes with vanishing N_f D6-branes lead to the Figures 5B and 10B of [19] with a rotation of $NS5'_L$ -brane by $\frac{\pi}{2}$ angle respectively.

Finally, the Figures 5B and 10B with a replacement N_f D6-branes by the NS5'-brane, when we neglect the $NS5'_R$ -brane, N''_f D6-branes and N''_c D4-branes and N'_f D6-branes, lead to the Figures 4B and 10B of [19] respectively.

For the same gauge groups in this paper, one can add an orientifold 6-planes together with three more NS-branes. Totally there exist seven NS-branes. The relevant previous works are given in [56, 57, 58, 50, 59]. Depending on the matter contents, there are two possibilities. On the other hand, for the different triple product gauge groups, one can add an orientifold 6-planes together with two more NS-branes. Totally, there exist six NS-branes. It would be interesting to study these meta-stable brane configurations in type IIA string theory.

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